Looking Back: Quantitative Analysis Before Computers

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A few security analysts engaged in quantitative analysis long before computers became readily available. This study presents an example dating from 1972. An analyst examined the fixed cost–variable cost structure of General Motors by creating a nonlinear algebraic model, transforming the model into a two-variable linear form, and estimating the model by means of a scatter diagram. The analyst's conclusions stand up remarkably well according to regression analysis 30 years after the fact. [G10, G12, G14]

■Before computers, only a few investment professionals engaged in quantitative analysis. One of us remembers performing the computations for a three-variable regression on an electromechanical calculator. It seemed to take forever and there was no assurance the result was error-free. Two-variable regressions were typically accomplished by means of scatter diagrams, eliminating tedious and impractical computations, and providing more clarity as well. We present an example of early quantitative analysis of General Motors in 1972. It was to estimate GM's fixed cost–variable cost structure. The steps were as follows.

- A nonlinear algebraic model was developed for GM's profit, assuming a fixed cost structure.
- The nonlinear profit model was transformed into a two-variable linear model involving ordinary financial margins and ratios.
- The two-variable linear model was fitted to the data using a scatter diagram. The fit was by eye.
- The scatter diagram was examined to see if there were evidence of cost structure changes over the period.
- The two-variable linear model was fitted separately to two halves of the data period. The fit was by eye.

The result was a reasonable estimate of GM's cost structure for each subperiod.

Our presentation presents the original analysis with a few wording changes to improve clarity. Then we examine the "fit by eye" accuracy. We conclude by updating the analysis to the present to assess whether the original analysis was indicative of the future.

I. 1972 General Motors Analysis

A. Model

In a well-managed company with a stable business, profit margin should be related to operating rate. A high operating rate should be associated with a high profit margin and a low operating rate with a low profit margin. This relationship can be used to estimate profit based on the operating rate. As only two variables are involved, a possible relationship can be investigated using a scatter diagram.

Suppose a company whose capacity and technology are fixed and whose market is stable. Profit, P, is the difference between sales, S, and cost, C:

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$$P = S - C \tag{1}$$

Cost can be separated into fixed and variable costs, C_F and C_V:

$$C = C_V + C_F \tag{2}$$

Fixed costs may be difficult to identify directly, but they should be related to the size of the firm. One measure of size is unit capacity, the maximum possible rate of unit output. Assume that fixed cost is proportional to unit capacity. Denote unit capacity by K_U and fixed cost per unit of unit capacity by c_{FU}. Fixed cost can then be modeled as:

$$C_F = c_{FU} K_U \tag{3}$$

Variable costs are equally difficult to identify directly, but they should be proportional to unit volume. Denote unit volume by U and variable cost per unit by c_{VU}. Variable cost can then be modeled as:

$$C_V = c_{VU} U \tag{4}$$

Equations (3) and (4) work fine if unit capacity and unit volume are available, although sometimes they are not. If unit volume is unavailable, it can be modeled by relating it to dollar sales. Assuming that price is fixed, sales is proportional to unit volume through price, s_U:

$$S = s_U U \tag{5}$$

Substitute Equations (3) and (4) in Equation (2). Then substitute the result into Equation (1):

$$P = S - C_V - C_F \tag{6}$$
$$P = s_U U - c_{VU} U - c_{FU} K_U \tag{7}$$
$$P = (s_U - c_{VU})\, U - c_{FU} K_U \tag{8}$$

The quantity $(s_U - c_{VU})$ on the right-hand side of Equation (8) is the incremental profit per unit.

The profit margin, M_p, is obtained by dividing Equation (8) by sales:

$$M_P \equiv \frac{P}{S} = (s_U - c_{VU})\left(\frac{U}{S}\right) - c_{FU}\left(\frac{K_U}{S}\right) \tag{9}$$

Now substitute from Equation (5):

$$M_P = \left(\frac{s_U - c_{VU}}{s_U}\right) - \left(\frac{c_{FU}}{s_U}\right)\left(\frac{K_U}{U}\right) \tag{10}$$

The first term in the parentheses on the right-hand side of Equation (10) is the incremental profit per unit divided by the price per unit, which is the incremental profit margin. The second term in the parentheses is the fixed cost per unit of capacity divided by the price per unit. This is the fixed cost per dollar of sales at capacity operation, which is the firm's lowest possible fixed cost margin. The last term in the parentheses is the inverse of the operating rate. Thus, the last two terms constitute the fixed cost margin at the actual operating rate.

Denote the operating rate by O. Then Equation (10) can be rewritten as follows:

$$M_P = \left(\frac{s_U - c_{VU}}{s_U}\right) - \left(\frac{c_{FU}}{s_U}\right)\left(\frac{1}{O}\right) \tag{11}$$

Equation (11) shows that the profit margin is the difference between the incremental profit margin and the fixed cost margin. The fixed cost margin is low to the extent that the operating rate is high. But as the operating rate can never exceed 1, the fixed cost margin can never be lower than the second term in the parentheses. Thus, the profit margin can never reach the incremental profit margin.

The relationship between the profit margin and the operating rate is shown in Figure 1. It is not linear. Equation (11) can be transformed into linear form by thinking of the relationship as between the profit margin and the inverse of the operating rate.

$$Y = A + BX \tag{12}$$

$$Y \equiv M_P \tag{13}$$

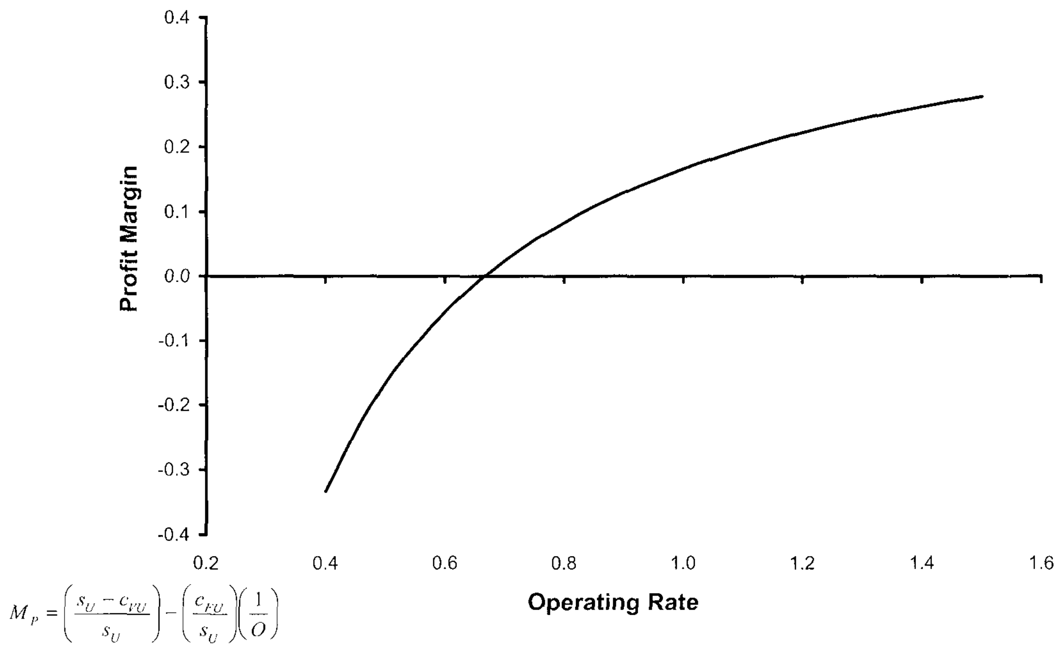
$$X \equiv \left(\frac{1}{O}\right) \tag{14}$$

$$A \equiv \left(\frac{s_U - c_{VU}}{s_U}\right) \tag{15}$$

$$B \equiv -\left(\frac{c_{FU}}{s_U}\right) \tag{16}$$

Thus, if Equation (11) is a reasonable model of the profit margin, a scatter diagram of profit margin against the inverse of the operating rate should result in an elongated pattern of points that can be adequately represented by a straight line. According to Equation (15), the line's intercept on the profit margin axis provides an estimate of the firm's incremental profit margin.

Figure 1. Relation Between Profit Margin and Operating Rate



According to Equation (16), the negative of its slope is an estimate of fixed cost per dollar of sales at capacity operation. Multiplying the latter estimate by dollar sales capacity, K_S, provides an estimate of fixed cost:

$$C_F = c_{FU} K_U = \left(\frac{c_{FU}}{s_U}\right) s_U K_U = \left(\frac{c_{FU}}{s_U}\right) K_S \tag{17}$$

A firm's operating rate may not be known accurately because unit capacity may not be known precisely. There is a way around this problem, which is to assume that unit capacity is proportional to the firm's size as measured by gross plant, G. Denote unit capacity per dollar of gross plant by k_{UG}. The model then takes the form:

$$K_U = k_{UG} G \tag{18}$$

Now the operating rate can be expressed as a function of sales per dollar of gross plant:

$$O = \frac{U}{K_U} = \frac{U}{k_{UG} G} = \frac{S/s_U}{k_{UG} G} = \left(\frac{1}{s_U k_{UG}}\right)\left(\frac{S}{G}\right) \tag{19}$$

This result can be substituted into Equation (11).

$$M_P = \left(\frac{s_U - c_{VU}}{s_U}\right) - \left(\frac{c_{FU}}{s_U}\right)\left(\frac{1}{O}\right) \tag{20}$$

$$M_P = \left(\frac{s_U - c_{VU}}{s_U}\right) - \left(\frac{c_{FU}}{s_U}\right)\left(\frac{s_U k_{UG}}{1}\right)\left(\frac{G}{S}\right) \tag{21}$$

$$M_P = \left(\frac{s_U - c_{VU}}{s_U}\right) - \left(c_{FU} k_{UG}\right)\left(\frac{G}{S}\right) \tag{22}$$

Using definitions as follows, this equation also is linear:

$$Y \equiv M_P \tag{23}$$

$$X = \left(\frac{G}{S}\right) \tag{24}$$

$$A \equiv \left(\frac{s_U - c_{VU}}{s_U}\right) \tag{25}$$

$$B = -\left(c_{FU} k_{UG}\right) \tag{26}$$

This result shows that the intercept of the line represented by Equation (22) is the same as before; only the slope has changed. The negative of the slope is the product of two quantities: 1) fixed cost per unit of capacity and 2) unit capacity per dollar of gross plant. The product is thus fixed cost per dollar of gross plant.[1]

B. Fitting the Model

We illustrate the model in Equation (22) using the General Motors data.

1. First Pass

Exhibit 1 presents a record of GM's operating profit margin and gross plant/sales ratio. The data are shown in the form of a scatter diagram in Figure 2. The points form a pattern well represented by the straight line that is fitted by eye. The model is realistic.

The line fitted in Figure 2 intercepts the profit margin axis at the value 42. According to Equation (25), this is an estimate of GM's incremental operating profit margin. An estimate of GM's quarterly fixed cost per dollar of gross plant is obtained by computing the negative of the line's slope.[2]

By definition, the slope of a line is the ratio of the vertical distance to the horizontal distance in going from one point on the line to another. Any two points on the line will give the same answer.

Two convenient points in Figure 2 are the ones directly above 0 and 4 on the gross plant/sales axis. The gross profit margins corresponding to these points are 42 and -4, respectively. From the point (0, 42) to the point (4, -4), the vertical distance traveled is (-4 - 42) = -46. The horizontal distance traveled is (4 - 0) = 4. The vertical distance needs to be divided by 100 because it is expressed in percentage form while the model is not. Thus the slope of the line is (-0.46/4) = -0.115.

The negative of the slope is 0.115. This is the estimate of the quarterly fixed cost per dollar of gross plant, expressed in dollars. Quarterly fixed cost per dollar of gross plant is estimated to be $0.115 or 11.5 cents.

These estimates make it possible to estimate the percentage of GM's costs that are fixed, at any desired operating level and size. Fixed cost is calculated by multiplying fixed cost per dollar of gross plant by gross plant. Variable cost is calculated by subtracting the incremental profit margin from 1.00 to get the incremental cost margin and then multiplying by quarterly sales.

As an example, for the first quarter of 1971 the company's sales were $7.765 billion and the gross plant $13.667 billion. Thus fixed cost was about (0.115 times 13.667) = $1.57 billion, and variable cost was about (1.00 minus 0.42) times 7.765 = $4.5 billion in that quarter. This suggests that variable cost represented about 74% and fixed cost about 26% of total cost.

2. Second Pass

The points in Figure 2 are labeled to identify the particular quarter. For example, the farthest right point, labeled 470, is for the fourth quarter of 1970.

A careful examination of the labels in Figure 2 shows that the points from the fourth quarter of 1968 (468) through the first quarter of 1971 (171) all lie below the line. Either this is a great coincidence, or the parameters of the model changed late in 1968. The nature of the auto industry suggests the latter hypothesis is the correct one. We can see the impact of these changes by fitting one line to the points from the first quarter of 1965 (165) through the third quarter of 1968 (368) and another line to the points from the fourth quarter of 1968 (468) through the first quarter of 1971 (171). Figure 3 graphs the results (lines fit by eye). The lower line represents the more recent period.

Both lines in Figure 3 have the same intercept. This means that the incremental profit margin and incremental cost margin were about the same in both periods, 39% and 61%, respectively. The line for the recent period, however, has a steeper downward slope. Evidently, the fixed cost per dollar of gross plant increased over time.

Using the same two points on the horizontal axis to measure the two slopes results in:

$$\frac{0.5-39.0}{4.0}=\frac{-38.5}{4.0}=-9.6 \qquad (27)$$

for the early period and

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[1]An easy way to see this and to avoid errors in this kind of work is through dimensional analysis. This is nothing more than keeping track of units by means of algebra. Write fixed cost per unit of capacity as:

$$\frac{fixedcost}{unitcapacity}.$$

Write unit capacity per dollar of gross plant as:

$$\frac{unitcapacity}{dollargrossplant}.$$

Multiply the two quantities and cancel unit capacity in the numerator and denominator:

$$\frac{fixedcost}{unitcapacity} x \frac{unitcapacity}{dollargrossplant} = \frac{fixedcost}{dollargrossplant}$$

The result is fixed cost per dollar of gross plant.

[2]Quarterly data are used.

Exhibit 1. General Motors Operating Profit Margin vs Gross Plant/ Sales Ratio

Year	Quarter	Label	Gross Plant/Sales	% Operating Profit Margin
1965	1	165	1.54	24.60
1965	2	265	1.57	23.50
1965	3	365	2.43	16.20
1965	4	465	1.63	22.70
1966	1	166	1.70	22.60
1966	2	266	1.81	20.60
1966	3	366	3.10	8.60
1966	4	466	1.83	20.80
1967	1	167	2.19	18.30
1967	2	267	1.95	20.70
1967	3	367	3.02	11.50
1967	4	467	1.92	21.10
1968	1	168	2.11	19.10
1968	2	268	1.82	20.80
1968	3	368	2.70	12.40
1968	4	468	1.75	20.00
1969	1	169	1.84	19.40
1969	2	269	1.95	17.90
1969	3	369	2.45	12.90
1969	4	469	1.93	17.90
1970	1	170	2.30	15.30
1970	2	270	1.99	16.70
1970	3	370	3.68	(0.60)
1970	4	470	4.57	(11.80)
1971	1	171	1.76	17.50

Figure 2. General Motors Profit Margin

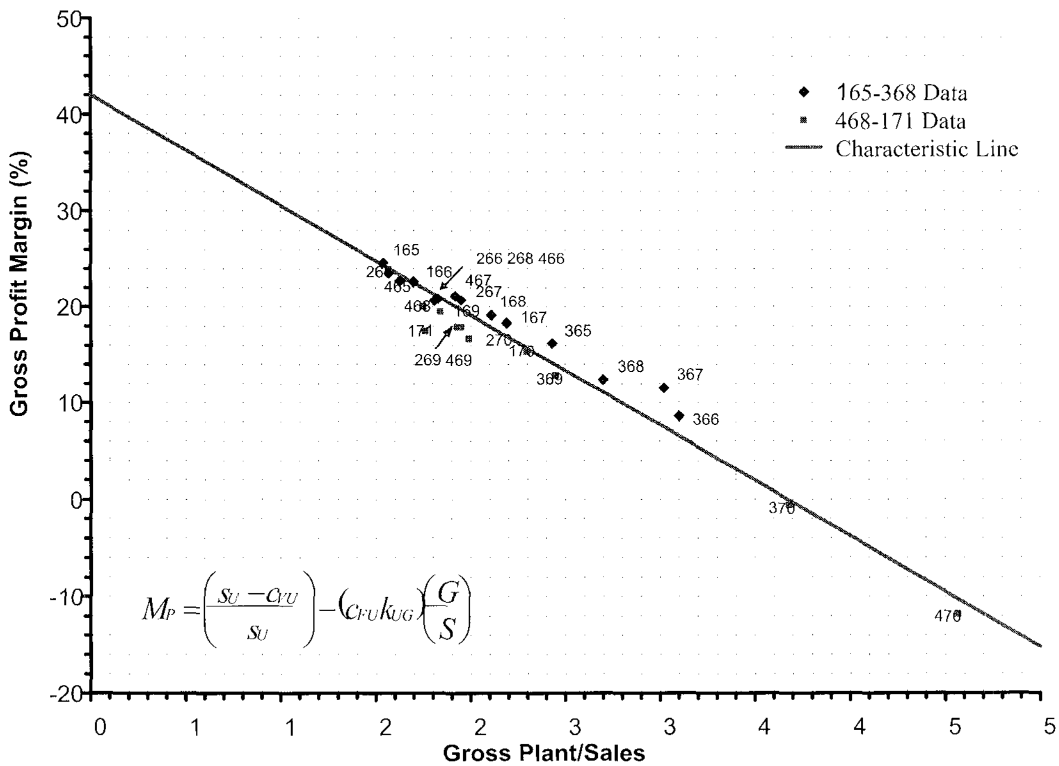
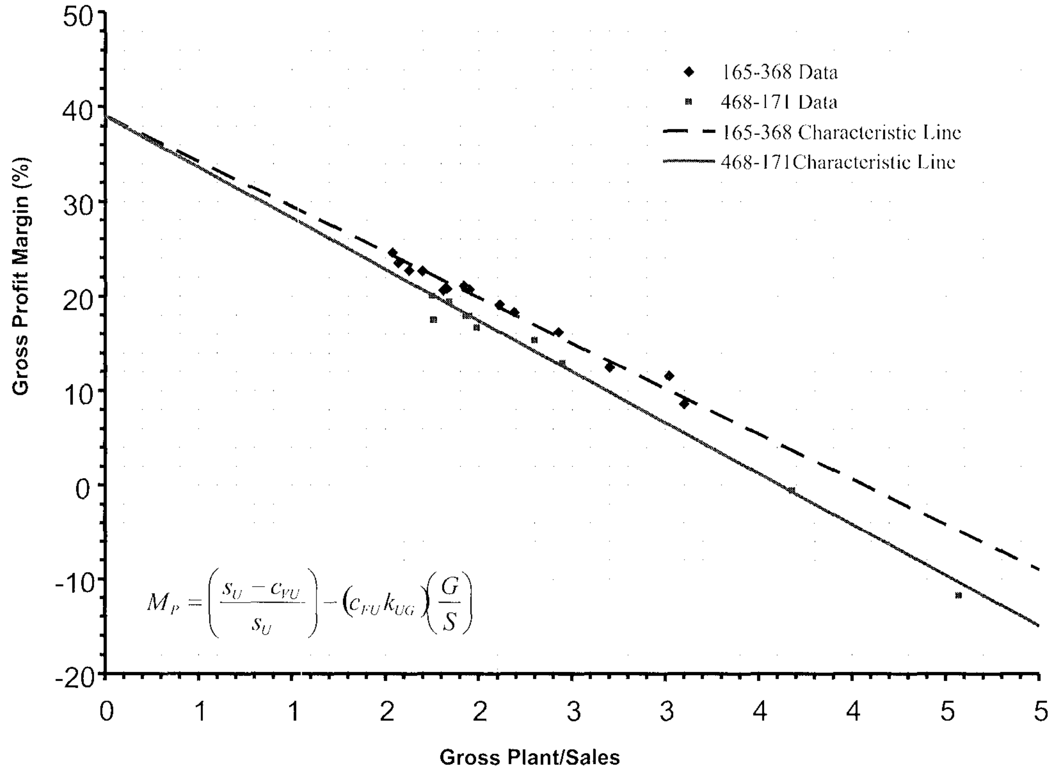


Figure 3. General Motors Profit Margin Comparisons: 1965-1968 vs. 1968-1971



$$\frac{-4.5 - 39.0}{4.0} = \frac{-43.5}{4.0} = -10.9 \tag{28}$$

for the more recent period. These correspond to quarterly fixed costs per dollar of gross plant of 9.6 cents in the earlier period and 10.9 cents in the later period.

It is interesting to see that the estimate of incremental profit margins obtained from Figure 3 is the same for both periods but lower than the estimate obtained from the pooled data in Figure 2. Another interesting observation is that both estimates of quarterly fixed cost per dollar of gross plant obtained from Figure 3 also are lower than the estimate obtained from Figure 2. This is contrary to intuition, which suggests that the estimates obtained from Figure 2 should be between the pairs of estimates obtained from Figure 3.

In fact, the intuition is wrong. If the premise behind Figure 3 is correct, then pooling of the data as in Figure 2 is inappropriate. Fortunately, the graph in Figure 2 with the 1968-1971 points below the line, made it clear that pooling might be inappropriate.

This example should indicate the importance of judgment and clarity in presentation in applying quantitative techniques. Models fitted to data for which they are not suitable can produce serious error.

II. Repeating the 1972 Data Analysis Using Regression

Was this analyst's "fit by eye" accurate? Would the analyst have reached the same conclusions using a formal regression analysis? The answers are yes to both questions.

Fitting the model in Equation (22) to the data in Exhibit 1 for the entire period, we obtain regression estimates of 40.497% for GM's incremental operating profit margin and 10.87 cents for quarterly fixed cost per dollar of gross plant. These estimates are very similar to the 42% and 11.5 cents the analyst obtained. The standard errors for the regression estimates are 1.164 and 0.500, respectively. This implies corresponding 95% confidence intervals of 38.090% –

42.904% and 9.837 cents – 11.903 cents. There is no material or significant difference between the fit by eye and the regression results.

The regression's adjusted R-square is 0.952; the standard error is 1.755; and the p-level for its F-value is 0.000.

We can use a dummy variable regression technique to check the analyst's conclusions about the mid-period change in GM's cost structure. The analyst's model in Equation (22) is of the form $Y = A + BX$. The regression coefficients A and B are modeled as follows:

$$A = A_1 + A_2 D$$
$$B = B_1 + B_2 D \quad (29)$$

where D is a dummy variable equal to 0 for the period 1/65 through 3/68 and 1 for the period 4/68 through 1/71. The new regression model is:

$$M_P = A + B\left(\frac{G}{S}\right) = \left(A_1 + A_2 D\right) + \left(B_1 + B_2 D\right)\left(\frac{G}{S}\right)$$
$$M_P = A_1 + A_2\left(D\right) + B_1\left(\frac{G}{S}\right) + B_2\left(D\frac{G}{S}\right) \quad (30)$$

Exhibit 2 presents the regression estimates compared to the early analyst's results. The regression R-square is highly significant, and the regression estimates for the two subperiods are remarkably close to the analyst's. The dummy variable is insignificant, suggesting that the incremental profit margin was unchanged. The product of the dummy variable and the gross plant/sales ratio is highly significant, suggesting that the fixed cost per dollar of gross plant changed, although not by much.

III. Extending the Analysis to the Present

To extend the analysis forward, we obtained quarterly data for GM's operating profit, gross plant, and sales from first quarter 1962 through second quarter 2003. Rolling six-year regression estimates of the incremental operating profit margin and quarterly fixed cost per dollar of sales using Equation (22) are plotted in Figure 4.

Following the analyst's period of analysis through 1972, there appears to be a downtrend in GM's incremental profit margin until about 1990. Then, beginning in about 1991, profitability rapidly recovered to the level estimated by the analyst. The only departure from the downtrend is a local maximum incremental operating profit margin at 1981.

The profitability downtrend appears to be related to GM's market share. Market share was about 46% in 1978 and declined to about 35% in 1991 (and to about 28% in 2002). GM's response in about 1990 was in the form of a massive downsizing, with other cost cutting and efficiency measures. This apparently is the reason the incremental profit margin returned to former levels between 1990 and 1993.

The same events explain the similar behavior of GM's estimated fixed cost per dollar of gross plant, which trended down to about zero and then rebounded to former levels. The fixed cost element corresponds to the negative of the slope of the operating profit margin – gross plant/sales ratio relationship. In other words, reported profit no longer depended strongly on the gross plant/sales ratio. One reason for the low estimates of fixed cost per dollar of sales is likely carrying unused plant on the books. For example, suppose that with loss of market share, GM idled half of its plants, but continued to carry them on the books and the original model specification applied to its remaining operations. In this case, the unused gross plant makes the gross plant/sales ratio used in the regression too high by a factor of two. But, since the original model specification is correct for the remaining operations, the net result is a fixed cost per dollar of gross plant estimate of one-half what it was.

These anomalous circumstances probably invalidate the model specification, as opposed to simply causing a structural break. If this is true, the regression estimates and their measures of significance are probably distorted. Yet the recovery shown in Figure 4 does suggest that events in the period from about 1983 through 1992 were nonrecurring.

We again fit a dummy variable regression model to the data for 1965 through second quarter 2003 to see if the analyst's 1965-1970 analysis was indicative of the future. The regression provides estimated changes (and their significance) in GM's incremental profit margin and fixed cost per dollar of gross plant for 1972 through second quarter 2003, measured relative to the 1965–1970 period:

$$M_P = A + B\left(\frac{G}{S}\right)$$
$$A \equiv A_1 D_1 + A_2 D_2 + A_3 D_3 + A_4 D_4 + A_5 D_5 + A_6 T \quad (31)$$
$$B \equiv B_1 D_1 + B_2 D_2 + B_3 D_3 + B_4 D_4 + B_5 D_5 + B_6 T$$

$D_i \equiv$ Five dummy variables that take the value of 1 for the periods 2/71-3/77, 4/77-1/84, 2/84-3/90, 4/90-1/97, and 2/97-2/03, and 0 otherwise. These provide estimated parameter shifts for the periods, measured relative to the 1/65-1/71 period.

$T \equiv$ A linear time variable to detect a time trend. This variable begins with a value of 13 for 1/65.

Exhibit 2. Regression of Operating Profit Margin vs. Gross Plant/Sales Ratio Test for Structural Shift in 1/65 to 1/71 Period

$$M_p = A_1 + A_2(D) + B_1\left(\frac{G}{S}\right) + B_2\left(D\frac{G}{S}\right)$$

Adjusted R Square	0.988
Standard Error	0.862
p value of F statistic	0.000

Variable	Estimate	Standard Error	p Value
Intercept	38.139	0.976	0.000
D	0.837	1.252	0.511
G/S	-9.214	0.455	0.000
D(G/S)	-1.709	0.547	0.005

Incremental Profit Margin	Regression Estimate	Fit By Eye Estimate
1/65 - 3/68	38.139	39.000
4/68 - 1/71	38.976	39.000
Fixed Cost/Dollar of Gross Plant		
1/65 - 3/68	9.214	9.600
4/68 - 1/71	10.923	10.900

Figure 4. Rolling Six-Year Regression Estimates: Operating Profit Margin (top line, left scale) and Quarterly Fixed Cost Per Dollar Of Sales (bottom line, right scale)

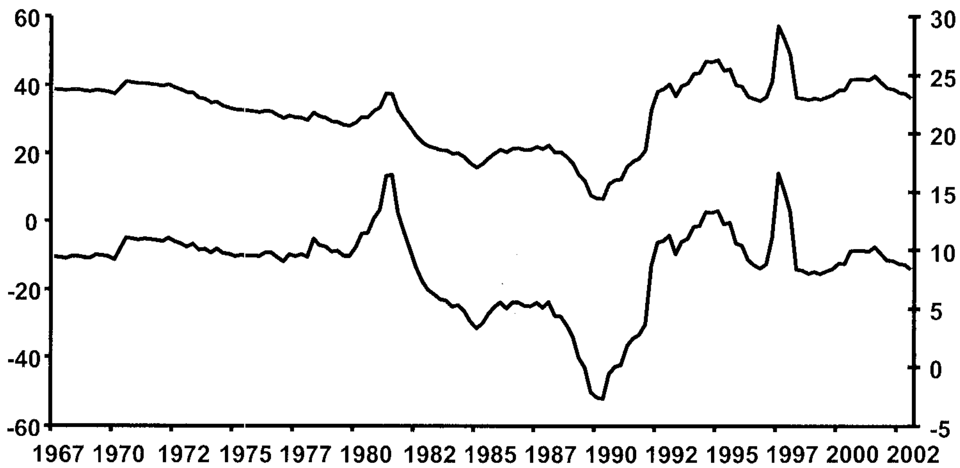


Exhibit 3. Analysis of Parameter Changes in 2/72 to 2/03 Period vs. 1/65 to 1/71 Period

Adjusted R-Square	0.391
Standard Error	4.932
p-value of F-statistic	0.000

Variable	Estimate	Standard Error	p Value
Intercept	49.202	7.712	0.000
D_1	-7.040	9.165	0.444
D_2	-7.848	11.392	0.492
D_3	-14.286	18.732	0.447
D_4	19.684	27.918	0.482
D_5	21.295	31.108	0.495
T	-0.258	0.246	0.297
G/S	-15.641	3.498	0.000
D_1G/S	-0.148	4.051	0.971
D_2G/S	-1.755	5.059	0.729
D_3G/S	2.270	8.369	0.787
D_4G/S	-11.886	11.639	0.309
D_5G/S	-13.165	12.831	0.307
T(G/S)	0.146	0.106	0.171

Incremental Profit Margin	Mid Period Regression Estimate	Fit By Eye Estimate
1/65 - 1/71	42.762	42.000
2/71 - 3/77	29.153	
4/77 - 1/84	21.648	
2/84 - 3/90	8.512	
4/90 - 1/97	35.785	
2/97 - 2/03	30.826	
Fixed Cost/Dollar of Gross Plant		
1/65 - 1/71	11.991	11.500
2/71 - 3/77	8.416	
4/77 - 1/84	6.226	
2/84 - 3/90	-1.594	
4/90 - 1/97	8.765	
2/97 - 2/03	6.320	

While this model provides more statistical detail about the period starting in the second quarter of 1972, it cannot compensate adequately for the dynamic nature of GM's operational changes and accounting procedures during its crisis years. For example, subperiod estimates could be misleadingly large. Consequently, the regression results should be treated as suggestive.

Exhibit 3 summarizes the results. The regression R-square, 0.391, is far below the 0.988 for the 1/65–1/71 period. The standard error, 4.932, is far higher than the earlier 0.862. The F statistic is highly significant. There probably is some value to the results.

In the second half of Exhibit 3, note that the regression estimates for the original period and the analyst's estimates are extremely close. There are no material or significant differences.

The dummy variable coefficients represent the increments to GM's estimated incremental operating profit margin for the subperiods, measured relative to the original period. Despite relatively wide fluctuations of the coefficient estimates, none of the dummy variables is significantly different from zero. Nor is there a strong indication of an underlying time trend.

The negative of the coefficient of the product of each dummy variable and the gross plant/sales ratio provides the estimated increment to GM's fixed cost per dollar of sales for various subperiods relative to the original period. Again none of these coefficients is significantly different from zero, nor is there a strong indication of an underlying time trend.

These two analyses provide insufficient statistical evidence to conclude that GM's fundamental operating relationships changed permanently after the first quarter of 1971. Considering events following the initial period of analysis, the analyst's estimates and conclusions hold up remarkably well.

IV. Conclusion

Some investment professionals were engaged in quantitative analysis well before the 1950s, preceding the computer age. An analyst in 1972 performed the equivalent of a two-variable regression of General Motor's operating profit margin against its gross plant/sales ratio by means of scatter diagrams. Fitting of a straight line to the points in the scatter diagram by eye produced estimates of GM's incremental profit margin and quarterly fixed cost per dollar of gross plant. We show that these estimates were virtually identical to those obtained from a regression analysis.

A separate question is whether either analysis would have been useful for forecasting. While GM began a financial decline shortly after the original analysis, the changes were not abrupt enough to foreclose the usefulness of the original analysis for prediction, especially given that the analyst's competitors had no way to accomplish anything as good. Moreover, GM's crisis years, as measured by its incremental operating profit margin and fixed cost per dollar of gross plant, appear to have been largely nonrecurring. Considering events following the original period of analysis, the estimates and conclusions held up remarkably well.■