NPV as a Function of the IRR: The Value Drivers of Investment Projects

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To enable better decision-making in capital budgeting, this article shows that the internal rate of return is a value driver of the project net present value, and that any free cash flow can be written as a function of some payback coefficient, the IRR itself, and time. Through the payback coefficients we can obtain a normalized rate of return that is completely compatible with the NPV. We also show that, in order to become compatible with NPV, the difference between the IRR and the opportunity cost of capital is to be discounted at the beginning of the period, because what matters is not the expected margin at the end of the period but instead its certainty equivalent at the beginning. [G31]

■ The debate in the finance literature about net present value (NPV) and the internal rate of return (IRR) as capital budgeting criteria concludes that the NPV is the best measure of the value created by a project under analysis (Brealey, Myers, and Allen, 2006). Yet at the same time, a variety of authors show the IRR is widely used among decision makers, mainly because it allows analysts to know the margin between the return of the project and the required return (see Bierman and Smidt, 2007). The IRR was once preferred by managers over the NPV [Gitman and Forrester (1977), Schall, Sundem and Geijsbeek (1978), and Jog and Srivastava (1995)], but Graham and Harvey (2001), and Brounen, de Jong, and Koedijk (2004) show a significant increase in use of the NPV. We aim to solve the discrepancy by showing that NPV and IRR are functionally related. A variety of consequences stem from this property. We first describe the functional relationship between NPV and IRR, which underlies the analysis of the value drivers of investment projects. Finally, we examine compatibility between analysis based on NPV and analysis based on rates of return by developing a method to obtain normalized rates of return.

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I. Functional Relation between NPV and IRR

The functional relation between the net present value and the internal rate of return that we develop is described for conventional projects, i. e., projects that present only one change of sign in their cash flow sequence. We later consider how our results apply to non-conventional projects. In the case of several initial capital outlays, we concentrate them, once properly capitalized, into one single cash flow that occurs at the beginning of the project.

We denote the cash flow sequence of the investment project under analysis by:

$$-c_0 \quad c_1 \quad \ldots \quad c_t \quad \ldots \quad c_n$$

To facilitate later development in the work, we need to express the initial capital outlay c_0 as a positive amount, therefore it is preceded by a negative sign.

The IRR is the rate of return embedded in the cash flow sequence of the investment project that makes the NPV equal to zero:

$$-c_0 + \sum_{t=1}^{n} \frac{c_t}{(1 + IRR)^t} = 0. \tag{1}$$

We can assign a non-negative proportion α_t of the initial capital outlay c_0 to any free cash flow c_t in such a way that the free cash flow c_t pays back the capital assigned $(\alpha_t c_0)$ and compensates it at the IRR:

$$\alpha_t \, c_0 \, (1 + IRR)^t = c_t. \tag{2}$$

The sum of these proportions is equal to one[1]:

$$\sum_{t=1}^{n} \alpha_t = 1. \tag{3}$$

We call α_t the payback coefficient of period t.

Let k be the opportunity cost of capital at which we discount future free cash flows. Then, the NPV is:

$$NPV = -c_0 + \sum_{t=1}^{n} \frac{c_t}{(1+k)^t}. \tag{4}$$

Now, we substitute c_t in this equation with its value formed in Equation (2):

$$NPV = -c_0 + \sum_{t=1}^{n} \frac{\alpha_t \cdot c_0 \cdot (1+IRR)^t}{(1+k)^t}, \tag{5}$$

which can be written as:

$$NPV = c_0 \cdot \left[-1 + \sum_{t=1}^{n} \alpha_t \cdot \left(\frac{1+IRR}{1+k}\right)^t\right]. \tag{6}$$

We thus obtain a function that relates the NPV and the IRR to one another, where the NPV is written as a dependent variable of the IRR. This equation reflects the fact that an IRR higher than the opportunity cost of capital makes a project feasible because its NPV is positive, but it also takes into account that the NPV depends on the distribution of cash flows, their size, and the life of the project. Thus, selecting from a set of mutually exclusive projects the project with the highest net return on capital, does not mean selecting the project with the highest NPV.

The NPV per monetary unit or normalized net present value (npv), which we use later, can be expressed as:

$$npv = \sum_{t=1}^{n} \alpha_t \cdot \left(\frac{1+IRR}{1+k}\right)^t - 1. \tag{7}$$

II. Value Drivers of an Investment Project

According to Copeland, Koller, and Murrin (2000), discounted cash flow drives the value of a company. The key value drivers of discounted cash flow are growth and the return on invested capital relative to the cost of capital. Both value drivers are implicit in the net present value, but our expressions make them explicit. Once we break the cash flow series into the size of the project, the payback coefficients, and the IRR, we can state that the NPV depends on the following value drivers:

- The internal rate of return.
- The time distribution of payback coefficients represented by the vector: $\{\alpha_1, \alpha_2, \ldots, \alpha_t, \ldots \alpha_n\}$.
- The opportunity cost of capital (k), which obviously subtracts value.
- The project's life (n).
- The investment size (c_0).

The ability of a project to create growth is expressed by its size and life, which are the value drivers that turn the IRR into monetary units. The return on invested capital relative to the cost of capital is expressed through the net rate of return η, which comes from the quotient between the capitalization factors of the IRR and the opportunity cost of capital:

$$\frac{1+IRR}{1+k} = 1 + \eta, \tag{8}$$

which we call net return on capital.

Substituting (8) into (6), NPV can be written as:

$$NPV = c_0 \cdot \left[-1 + \sum_{t=1}^{n} \alpha_t \cdot (1+\eta)^t\right]. \tag{9}$$

In this case, NPV is calculated by means of an equation that apparently includes a capitalization factor, $(1+\eta)^t$. Nevertheless, as (8) indicates, the term $(1+\eta)^t$ is, in fact, the outcome of having discounted $(1+IRR)^t$ at k. The normalized net present value becomes:

$$npv = \left[-1 + \sum_{t=1}^{n} \alpha_t \cdot (1+\eta)^t\right]. \tag{10}$$

From (8) we can write:

$$(1+k) \cdot (1+\eta) = 1 + IRR. \tag{11}$$

In Equation (11) we can interpret the net rate of return on capital η as the additional growth rate that is added to the opportunity cost of capital in order to reach a growth rate equal to the IRR. Solving (8) for η, we obtain:

$$\eta = \frac{IRR - k}{1+k}. \tag{12}$$

This result shows that the net return on capital does not consist of the difference between the internal rate of return and the opportunity cost of capital, but of the quotient of this difference by the capitalization factor of the opportunity cost of capital, $1 + k$. Therefore, a simple comparison of the difference between the IRR and the opportunity cost of capital of two mutually exclusive projects can be misleading as to which project generates a higher net return when both projects have different opportunity costs of capital, i.e., different required rates of return. This means that, even if two projects have the same size, life, and payback coefficients and can thus be compared exclusively on the basis of their net returns,

[1] The proofs can be obtained from the authors via email to joan.montllor@uab.es.

interpreting the difference IRR - k as the net return can result in choosing the project with a lower NPV.

The fact that the net rate of return consists of the difference between the normalized rate of return and the opportunity cost of capital, IRR - k, discounted at the opportunity cost of capital, can be interpreted by saying that both the IRR and the opportunity cost of capital are rates referring to the end of the period. Nevertheless, the difference between them, in order to be compatible with the NPV, has to be measured at the beginning of the period and, for this reason, discounted at the opportunity cost of capital.

What matters is not the expected difference at the end of the period, but rather its certainty equivalent at the beginning. This observation is particularly important for venture capital projects because of their usually very high required rate of return.

III. Rates of Return Equivalent to the NPV: Normalized Rate of Return

We have noted that empirical studies show that managers making capital budgeting decisions often prefer percentage measures to monetary unit measures, i.e., prefer the IRR over the NPV. Yet net present value incorporates the complete set of value drivers of the investment project, while the internal rate of return is just one of them. This makes it necessary to calculate rates of return that lead to the same decisions as the NPV. Logically, these rates do not present Fisher's intersections.

For projects with the same size and life, heterogeneity among vectors of payback coefficients is what makes the net return on capital η less useful ranking projects than the NPV. We address this drawback by developing a method that first establishes equivalence of projects under analysis and a set of projects with homogeneous payback coefficients chosen by the analyst, and then provides a rate of return that ranks projects according to the NPV. We call the rates of return obtained under this equivalence, *normalized rates of return*. Once the NPV of the project is known, we can obtain a new IRR that fits with a new vector of payback coefficients and this NPV[2].

Let us suppose a firm wants to turn the cash flow series of a specific investment project into the one that determines this new vector of payback coefficients. The firm would reshape the project by reinvesting at the opportunity cost of capital the cash flows it wants to delay, and by raising funds at this rate for the cash flow advances required by the reshaping of the project. The vector of payback coefficients and its corresponding cash flows have changed after this operation, but the net present value of the project has not.

Let A be an investment project. We define its equivalent normalized investment project A^* for a specific net present value as a project of the same size, life, and NPV, whose payback coefficients are equal to the vector that the analyst has decided to use in order to normalize the set of projects under analysis.

We write this vector, the normalizing vector, as:

$$\{\alpha_1^*, \alpha_2^*, ..., \alpha_t^*, ..., \alpha_n^*\}. \tag{13}$$

Thus, the normalized project fulfills four conditions:

1) Equivalent size:

$$c_{0A^*} = c_{0A}; \tag{14}$$

2) Equivalent life:

$$n_{A^*} = n_A; \tag{15}$$

3) Equivalent net present values:

$$NPV_{A^*} = NPV_A; \text{ and} \tag{16}$$

4) A vector of payback coefficients that is equal to the normalizing vector, which implies:

$$\alpha_{At}^* = \alpha_t^*; \tag{17}$$

where α_{At}^* denotes the payback coefficient of the normalized project at period t.

Once we calculate the NPV of investment project A, we can calculate the IRR that it's equivalent normalized investment project A^* should earn in order to have the same NPV for the same opportunity cost of capital. We call it *normalized rate of return* (*NR*). Taking (6) into account, the equality between the NPV of the original project and the NPV of the equivalent normalized investment project A^* means that:

$$c_0\left[-1 + \sum_{t=1}^{n}\alpha_t^*\left(\frac{1+NR}{1+k}\right)^t\right] = c_0\left[-1 + \sum_{t=1}^{n}\alpha_t\left(\frac{1+IRR}{1+k}\right)^t\right]. \tag{18}$$

Thus, the *NR* has to fulfill the equality:

$$\sum_{t=1}^{n}\alpha_t^* \cdot \left(\frac{1+NR}{1+k}\right)^t = \sum_{t=1}^{n}\alpha_t \cdot \left(\frac{1+IRR}{1+k}\right)^t, \tag{19}$$

[2] Specific sets of payback coefficients are constant payback coefficients and the sets that turn the original project into constant cash flows and the zero-coupon project. The normalized rate of return provided by the latter is equal to the modified rate of return defined by Bernhard (1989) after Lin (1976), often called the modified internal rate of return. Bernhard (1989) rightly drops the adjective *internal*, because it depends on the reinvestment rate, i.e., an external rate. Chang and Swales (1999) describe the advantages of the modified rate of return over the IRR.

Exhibit 1. Numerical Illustration

This table shows a numerical illustration of the analysis presented in the article for mutually exclusive projects A, B, and C. Because of the multiple values of its IRR, project C has no payback coefficients. NPV recommends project A; but, because of differences in the payback coefficients of projects A and B and the multiple IRR of project C, in order to find a rate of return that fits with NPV, we have to calculate the normalized net rate of return, η^*, after having normalized the projects.

Panel A. Cash Flows

Period		0	1	2	3	4
Project	k					
A	7.50 %	-1000	115	264.50	608.35	524.70
B	20.00 %	-1000	390	507.00	549.25	428.42
C	20.00 %	-1000	8550	-24875.00	29850.00	-12600.00

Panel B. Payback Coefficients

Period	0	1	2	3	4
A		0.10	0.20	0.40	0.30
B		0.30	0.30	0.25	0.15

Panel C. Profitability Measures

	NPV	IRR	η
A	218.45 $	15%	6.98%
B	201.54 $	30%	8.33%
C	48.61 $	5%; 50%; 100%; 300%	

Panel D. Equivalent Normalized Projects
Normalizing Vector: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.25$
Cash Flows of Normalized Projects

Period	0	1	2	3	4
A	-1000	290.41	337.36	391.90	455.26
B	-1000	322.44	415.88	536.38	691.81
C	-1000	305.72	373.87	457.20	559.10

Panel E. Profitability Measures of Equivalent Normalized Projects

	NPV	NR	η^*
A*	218.45 $	16.16 %	8.06 %
B*	201.54 $	28.98 %	7.48 %
C*	48.61 $	22.29 %	1.91 %

from which the *NR* can be calculated by numerical methods.

From the normalized rate of return and the opportunity cost of capital, we obtain the normalized net rate of return on capital (η^*):

$$\frac{1 + NR}{1 + k} = 1 + \eta^*. \tag{20}$$

Using Equations (7), (19), and (20) we can write the normalized net rate of return on capital as a function of the normalized net present value (npv):

$$npv = \sum_{t=1}^{n} \alpha_t^* \left(1 + \eta^*\right)^t - 1. \tag{21}$$

Equation (21) shows that *npv* and η^* produce identical rankings for the set of projects under analysis, because all these projects have the same normalized payback coefficients.

The role of the normalized rate of return *NR* is to provide decision makers with a rate of return that, once turned into a net rate, becomes systematically equivalent to NPV for projects of the same size and life. Yet there is an essential difference between the original IRR and the *NR*; the former is a value driver, i.e., an independent variable of the function of the NPV, and the latter is a dependent variable of the NPV. The normalized rates of return are not primary measures of profitability, because they have not been obtained directly from the investment project but rather have required the previous calculation of its NPV. The objective of the NR is to obtain a margin that represents a reliable reference for ranking projects by sharing the clarity of a margin and the reliability of the NPV.

The normalized rate of return *NR* and its corresponding net rate η^* can be directly applied to non-conventional projects because they are independent of the cash flow sequence of the original project. Thus, the method of ranking projects using the NPV and the normalized net rate of return does not require any change for unconventional projects.

Exhibit 1 provides a numerical illustration of this procedure.

IV. Conclusion

Capital budgeting requires a complete identification of the value drivers of the net present value. We contribute to this process by showing that writing the NPV as a function of the IRR instead of a cash flow series helps to identify as the value drivers of investment projects: the IRR itself; the opportunity cost of capital; the set of payback coefficients that, through the IRR, links the amount invested with the free cash flow series of the project; project size; and project life.

The net rate of return consists of the discounted value of the difference between the IRR and the opportunity cost of capital, not simply the difference itself. Equivalence of the sets of payback coefficients is a necessary condition for ranking mutually exclusive projects of the same size and life through the net rate, because, being payback coefficients the unique value driver in which they differ, once we normalize them the NPV and the net rate provide the same ranking.

Any set of projects under analysis can be transformed into a new set where all projects have equal payback coefficients and, at the same time, maintain their original NPV. From these normalized projects we obtain a normalized net rate of return that ranks projects of the same size and life as the NPV does. The normalized rate of return, however, is not a value driver or a cause of the NPV, but rather a consequence of it, because it is obtained from the NPV after making homogeneous the payback coefficients of the set of projects under analysis. ■

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