The Speed of Stock Price Discovery

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Abstract

This paper develops closed-form expressions for the path and speed of stock price discovery in a simple asset allocation model. The representative investor is composed of two uniquely defined investors whose different risk-preferences always generate opposite portfolio rebalancing trades. The implied supply and demand schedules for shares determine the intra-period path and speed of price discovery in a tâtonnement setup. Convergence to equilibrium is exponential, and its speed depends on information content, risk-preferences, firm size, the market price for risk, and the price impact of excess demand. Convergence is not guaranteed, and the conditions for divergence are specified.

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1. Introduction

The efficient market hypothesis (Fama, 1970) defines a market as being informationally efficient if the financial assets' prices reflect the value of new information "fast enough", such that investors cannot exploit news to systematically earn excess returns. Yet asset pricing theories, in particular the CAPM of Sharpe (1964) and Lintner (1965), and the Arbitrage Pricing Theory of Ross (1976), offer no explicit account for the duration of price discovery. Bossaerts (2002) appears to be the first to establish a model analyzing the path (not speed) of price discovery in a CAPM setup. This model, later expanded by Asparouhova et al. (2003), builds on principles of general equilibrium theory by applying a Walrasian clearing [i.e., excess demand (supply) induces price increase (decline)] in a tâtonnement setup.[1] Asparouhova et al. demonstrate that if convergent, the price path is exponential, and the conditions for a divergent path are studied.[2]

The model developed in this paper extends Asparouhova et al.'s analysis in a few aspects. First, we provide a closed-form expression for the *speed* of price discovery in units of time. Second, in addition to specifying the beginning- and end-of-period optimum asset holdings, as Asparouhova et al. do, we provide closed-form specifications for the period supply and demand schedules for shares. These functional forms determine the path and speed of price discovery, as well as whether the path is convergent or divergent. Third, we show that it is sufficient to

[1] The literature on dynamic stability of a tâtonnement process, where exchange takes place only at the equilibrium price, have been studied extensively (e.g., Negishi (1962) and Arrow and Hahn (1971)). Hahn and Negishi (1962) analyze stability properties of non- tâtonnement processes, where trade occurs throughout the price discovery path.

[2] Bossaerts et al. (2000) discuss theoretical and empirical aspects of price discovery in a CAPM framework, and present experimental results. Meloso et al. (2009) present experimental evidence that the speed of price discovery in an anonymous rights market is faster than that of a patent system.

distinguish between two investor types on the premise of attitude toward risk in order to motivate trade in the CAPM. Lastly, we show how the speed and path of price discovery are determined by the primitives of the asset pricing system, particularly differences in risk preferences, as well as by the market capacity in processing excess demand. We specify the typical system time scale unit and find that the speed of price discovery increases with firm size, average risk aversion, and standard deviation of the risky asset, but decreases with the equity premium.

The study of the *speed* of stock price discovery has until now been addressed empirically and in laboratory experiments (see footnote 2). The early empirical tests include Epps (1979), Hillmer and Yu (1979), and Patell and Wolfson (1984). The evidence suggested that price adjustments are not immediate, and profitable trades are plausible. Patell and Wolfson (1984) report that the initial stock price response to news (dividend and earning announcements) is evident a few minutes after the news release and in most cases profit opportunities last 10 minutes after the first response. Extreme news, changes in variance, and changes in serial correlation of returns may slow the speed of price discovery, sometimes until the following opening. A number of papers expanded the analysis in various aspects, among them Jennings and Starks (1985, 1986), who condition on informational content and on the existence of tradable options on shares. They found that the speed of price adjustment is faster when the information content is low and when options are written on the examined stocks. Gosnell et al. (1996) report slower speed of adjustment to substantial changes in dividend policy as compared to Patell and Wolfson, and further report that bad news takes more time (up to 75 minutes) than good news to be fully absorbed in prices. Our model predicts faster price discovery when the information content is low, consistent with the empirical findings.

The specific characteristics of market microstructure and the technology applied to market clearing, together with the pace of information dissemination across investors, also appear to be relevant for the speed of price discovery. Martens (1998) explores the speed of price discovery of German Bund futures contracts trading at two exchanges: the Deutsche Terminborse (DTB) which applies an electronic trading system, and the London International Financial Futures Exchange (LIFFE), employing floor trading. He finds that the speed of price discovery is faster in LIFFE in volatile periods, arguing that in a fast moving market canceling orders in the electronic book is time consuming. Evidence that improved information availability, lower trading costs, and technological improvements of trading mechanisms increase the speed of price discovery is given by Busse and Green (2002). They show that positive news on stocks discussed by analysts at CNBC affected the relevant stock prices within seconds, with the effect lasting approximately one minute. Yet the price response to negative news was more gradual, lasting about 15 minutes, possibly due to higher short-selling costs. Additional evidence to that effect is given by Frijns and Schotman (2005), demonstrating that within 1-5 minutes following a quote innovation the tick-time speed of price discovery converges exponentially to a steady state. This finding is consistent with Asparouhova et al. (2003) and with our model, as the path of price discovery following an information shock is exponential in both models.

Madhavan and Panchapagesan (2000) analyze the role that specialists play in speeding price discovery in a single-price auction market model. Using NYSE opening data, they show that specialists speed the opening price discovery as they extract valuable information from the book. Similarly, Chordia et al. (2005) found that positively autocorrelated order imbalances stemming from impatient investors induce NYSE specialists to revise quotes away from fundamental prices, in order to control inventory. They argue that arbitrageurs submit counter trades so that the order imbalance can be closed within a few minutes. Garvey and Wu (2008) compare order execution cost versus speed throughout the trading day in NASDAQ, and show that while executions are faster near the opening and close than around midday, they are also more expensive.

While empirical evidence on the speed of price discovery abounds, there are fewer theoretical models of the subject, and with the exception of Bossaerts (2002) and Asparouhova et al. (2003), all are in the market microstructure domain. The earliest related paper appears to be Mendelson (1982), who analyses properties of a sealed-bid, double auction market that clears every T unit of time throughout a trading period. While Mendelson analyzes volume and liquidity throughout the interval T, there is no explicit account for the speed of the clearing process in calendar time. Essentially, Mendelson's model considers clearing as a matching process, and like Copeland and Galai (1983), Glosten and Milgrom (1985). and Kyle (1985), the specific timing of order arrival is exogenous and irrelevant for price determination. A few subsequent microstructure models, starting with Diamond and Verrecchia (1987) and Easley and O'Hara (1987, 1992), endogenize the orders' time of arrival and analyze the strategic trading consequences of signals embedded in no-trade intervals. Easley et al. (2008) interpret the calendar speed of price discovery across different levels of PIN (proportion of informed trades) as the slope of the price impact function (ibid, Figure 4), and find that speed increases with PIN.

As mentioned, Bossaerts (2002) and Asparouhova et al. (2003) were the first to formally explore the stock price discovery in a CAPM setup. More specifically, they consider a static model of a riskless bond and many risky assets, with uniformly endowed Constant Absolute Risk Aversion (CARA) investors who differ in the parameter of risk aversion. The path and stability of price discovery are analyzed in a tâtonnement setup as the portfolio composition changes from the beginning-of-period asset holdings to the end-of-period holdings. In our model, myopic investors solve the next-period asset allocation problem between a riskless and a risky asset. There are two investor types with different endowments: one has Constant Relative Risk

Aversion (CRRA) and the other Decreasing Relative Risk Aversion (DRRA). Investors' differences in risk preferences imply opposite portfolio rebalancing rules – one yields a supply schedule for shares and the other a demand schedule. These schedules facilitate the analysis of the path, stability, and speed of price discovery. The price path is exponential, whether it is convergent or divergent. Convergence to the equilibrium price depends on relative endowments between the two investor types, as well as asset allocation within each investor's portfolio. Once the proportion of DRRA investors surpasses a threshold that we specify in closed-form, the price path turns divergent.

A description of the economy and the derivation of the optimal asset allocation rules are presented in Section 2. We derive the supply/demand schedules for shares in Section 3, study the speed of price discovery in Section 4, and calibrate and estimate it with NYSE postwar data in Section 5. Section 6 summarizes the paper.

2. Optimal Asset Allocation

Our goal is to analyze the speed of stock price discovery from the stock equilibrium price P_t to the equilibrium price $P_{t+\Delta t}$ in discrete time, following Ross (1975) and Friend and Blume (1975), who solve for the optimal asset allocation with myopic, heterogeneous investors. Using a similar construct, we explore how price changes induce rebalancing trades by the two myopic investors indexed $k = (a, b)$. We assume that both investors have different parameter values of the Hyperbolic Absolute Risk Aversion (HARA) utility function

$$U_k(W_k, t) = e^{-\rho_k t} \frac{\delta_k}{1-\delta_k} \left(\frac{W_k}{\delta_k} + \eta_k \right)^{1-\delta_k} .$$

Here δ captures relative risk aversion while η is a displacement parameter. The financial markets are assumed frictionless where a single riskless bond is available at infinite supply and a single (representative) stock is available at a given supply of N_t shares at *t*. The rate of return on the bond is *r*, the expected rate of return on the stock is μ, and σ is its standard deviation. Define the proportional allocation of total wealth $\overline{W}_{k,t}$ between both assets as $\alpha_{k,t} = N_{k,t} P_t / \overline{W}_{k,t}$ to the stock, where $N_{k,t}$ is number of shares, and $1 - \alpha_{k,t} = Q_{k,t} B_t / \overline{W}_{k,t}$ to the bond, where $Q_{k,t}$ is quantity of bonds and B_t their price. Let $\Delta P_{t+\Delta t} / P_t = \mu \Delta t + \sigma z_t \sqrt{\Delta t}$ and $\Delta B_{t+\Delta t} / B_t = r \Delta t$, which imply that investor *k*'s next period wealth conserving budget constraint is

$$\begin{aligned} \hat{W}_{k,t+\Delta t} &= N_{k,t} P_{t+\Delta t} + Q_{k,t} B_{t+\Delta t} \\ &= \overline{W}_{k,t} \left(1 + (r + \alpha_{k,t}(\mu - r))\Delta t\right) + \alpha_{k,t} \overline{W}_{k,t} \sigma z_t \sqrt{\Delta t} \end{aligned}, \tag{1}$$

where $\hat{W}_{k,t+\Delta t}$ is period $t + \Delta t$ wealth before rebalancing, $\overline{W}_{k,t}$ is period *t* wealth after rebalancing, and $\hat{W}_{k,t+\Delta t} = \overline{W}_{k,t+\Delta t}$ holds by the property of wealth conservation. Being myopic, investors' maximization problem is $Max\, E\left[U(\overline{W}_{k,t+\Delta t}) \middle| \overline{W}_{k,t}\right]$. A standard Taylor series expansion about $\hat{W}_{k,t+\Delta t}$ yields the optimal asset allocation rule at *t*

$$\alpha^*_{k,t} \overline{W}_{k,t} = \frac{\mu - r}{\delta_k \sigma^2} \left(\overline{W}_{k,t} + \eta_k \delta_k\right).$$

To derive the optimal portfolio rebalancing policy by investor *k*, one must specify the optimum asset allocation both in period *t* and in period $t + \Delta t$. Incrementing the time index, the optimal rule at $t + \Delta t$ is

$$\alpha^*_{k,t+\Delta t} \overline{W}_{k,t+\Delta t} = \frac{\mu - r}{\delta_k \sigma^2} \left(\overline{W}_{k,t+\Delta t} + \eta_k \delta_k\right). \tag{2}$$

Based on the property of wealth conservation, replace $\overline{W}_{k,t+\Delta t}$ with $\hat{W}_{k,t+\Delta t}$ in the right-hand-side of (2), and express asset values in terms of prices and quantities as in (1),

$$N_{k,t+\Delta t} P_{t+\Delta t} = \frac{\lambda}{\delta_k}\left(N_{k,t} P_{t+\Delta t} + Q_{k,t} B_{t+\Delta t} + \eta_k \delta_k\right), \tag{3}$$

where $\lambda \equiv (\mu - r)/\sigma^2$. The number of shares held at $t + \Delta t$ may be written as $N_{k,t+\Delta t} = N_{k,t} + \Delta N_{k,t+\Delta t}$. Replace it with $N_{k,t+\Delta t}$ on the left-hand-side of (3) and solve for a hypothetical stock price $P^H_{t+\Delta t}$, satisfying $P^H_{t+\Delta t} = P_{t+\Delta t}$ in equilibrium. The functional relationship between quantity traded and the hypothetical price $P^H_{t+\Delta t}$ is

$$P^H_{t+\Delta t} = \frac{\frac{\lambda}{\delta_k}\left(Q_{k,t} B_{t+\Delta t} + \eta_k \delta_k\right)}{N_{k,t}\left(1 - \frac{\lambda}{\delta_k}\right) + \Delta N_{k,t+\Delta t}}, \tag{4}$$

which may be increasing or decreasing in the ($P^H_{t+\Delta t}$, $\Delta N_{k,t+\Delta t}$) plane, depending on the values of δ_k and η_k, given λ, and asset allocation at *t*. Equation (4) is similar in essence to equation 3 in Johnson (2006), where the substitution between quantities and prices are derived, such that the investor's optimum policy remains intact.

3. "Supply" and "Demand" Schedules[3]

The following Proposition distinguishes between two schedules of positive and negative rebalancing trades given a price change, based on the specific values of δ and η. In the following section we analyze the process of price discovery, based on these schedules.

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[3] The terms "supply" and "demand" are in double quotation marks since both supply and demand for shares are exhibited along each of the curves, depending upon whether the price change is positive or negative. Because the former is rightward increasing and the latter decreasing, resembling supply and demand functions, we use the terms "schedules" "supply", and "demand" interchangeably.

Proposition: *The slope of (4) is positive iff* $\left\{\begin{array}{l}\eta_a < -Q_{a,t}B_{t+\Delta t}/\delta_a\big|_{Q_{a,t}>0} \\ \delta_a < \lambda\end{array}\right\}$*, or negative iff* $\left\{\begin{array}{l}\eta_b > -Q_{b,t}B_{t+\Delta t}/\delta_b\big|_{Q_{b,t}>0} \\ \delta_b > \lambda\end{array}\right\}$.

Proof:

<u>A. Positive slope (investor type "a")</u>: A positive slope holds when $\frac{\partial P^H_{t+\Delta t}}{\partial(\Delta N_{a,t+\Delta t})} > 0$*,*

$$\frac{\partial P^H_{t+\Delta t}}{\partial(\Delta N_{a,t+\Delta t})} = -\frac{\frac{\lambda}{\delta_k}\left(Q_{a,t}B_{t+\Delta t} + \eta_a\delta_a\right)}{\left(N_{a,t}\left(1-\frac{\lambda}{\delta_a}\right) + \Delta N_{a,t+\Delta t}\right)^2} > 0\,;$$

which implies that $Q_{a,t}B_{t+\Delta t} + \eta_a\delta_a < 0$ *should hold. If agent a is net lender (*$Q_{a,t} > 0$*) and risk averse (*$\delta_a > 0$*), the numerator is negative when* $\eta_a < -Q_{a,t}B_{t+\Delta t}/\delta_a$*.*

<u>B. Negative slope (investor type "b")</u>: A negative slope in (4) holds iff $\eta_b > -Q_{b,t}B_{t+\Delta t}/\delta_b$*, the investor is risk averse (*$\delta_b > 0$*) and a net lender (*$Q_{b,t} > 0$*).*

<u>C. The boundaries of δ_k</u>: To prove the boundaries of δ_k *solve (3) for* λ/δ_k *,*

$$\frac{\lambda}{\delta_k} = \frac{N_{k,t+\Delta t}P_{t+\Delta t}}{N_{k,t}P_{t+\Delta t} + Q_{k,t}B_{t+\Delta t} + \eta_k\delta_k}\,.$$

If a positive slope holds, $Q_{a,t}B_{t+\Delta t} + \eta_a\delta_a < 0$*, which implies that the inequality* $\delta_a << \lambda$ *should be strong enough as* $N_{k,t+\Delta t}/N_{k,t}$ *fluctuates about unity; the opposite holds for a downward sloping curve, where* $\lambda << \delta_b$ *must hold.* *Q.E.D.*

The Proposition implies that if investors' preferences are segmented such that one has a positive slope and the other a negative slope, there will be one investor type whose implied optimal trade is positive (buy shares) given price increases but negative given price declines. The other investor trades in opposite directions given same price changes. Because implied trades are determined by preferences, expectations, and endowments, and because the market must clear, we may analyze the properties of the intra-period process of price discovery as equilibrium prices are changing from P_t to $P_{t+\Delta t}$.[4]

4. The Speed of Price Discovery

Time within the period interval Δt will be indexed henceforth by τ. It is assumed that throughout the tâtonnement process the auctioneer calls new prices at regular time intervals $d\tau$. We mention that there is no exchange throughout the period, thus the period *t* asset allocation remains the relevant determinant of supply and demand schedules throughout the tâtonnement process. For notational simplicity define the following based on (4): $p_0 \equiv P_t$ is the equilibrium price at *t* and $p^* \equiv P_{t+\Delta t}$ is the equilibrium price at $t+\Delta t$; $q^{*d} \equiv \Delta N_{b,t+\Delta t}$, and $q^{*s} \equiv \Delta N_{a,t+\Delta t}$ are the quantities exchanged at the end-of-period; $q_\tau^d \equiv \Delta N_{b,t+\tau}$ and $q_\tau^s \equiv \Delta N_{a,t+\tau}$ are intra-period quantities revealed but not traded by both investors throughout the tâtonnement process. We further define: $a^d \equiv \frac{\lambda}{\delta_b}\left(Q_{b,t}B_{t+\Delta t} + \eta_b\delta_b\right)$, $a^s \equiv \frac{\lambda}{\delta_a}\left(Q_{a,t}B_{t+\Delta t} + \eta_a\delta_a\right)$, $b^d \equiv N_{b,t}\left(1 - \frac{\lambda}{\delta_b}\right)$, and $b^s \equiv N_{a,t}\left(1 - \frac{\lambda}{\delta_a}\right)$. Based on (4), the demand function (super-script *d*) is

[4] Constantinides (1982) shows that if all investors maintain Pareto optimal allocations, prices are determined as if a representative investor exists. Our simulations below guarantee that the decomposition of the representative investor is consistent with the equilibrium price.

$$q^{*d} = \frac{a^d}{p^*} - b^d\,, \tag{5}$$

and it is changing throughout the period according to

$$q_\tau^d = \frac{a^d}{p(\tau)} - b^d\,. \tag{5a}$$

Equivalently, the supply function (super-script *s*) is

$$q^{*s} = \frac{a^s}{p^*} - b^s\,, \tag{6}$$

and its intra-period version is

$$q_\tau^s = \frac{a^s}{p(\tau)} - b^s\,. \tag{6a}$$

The end-of-period equilibrium price p^* must be consistent with market clearing such that $q^{*d} = -q^{*s}$ (ignoring instantaneous share issue or buyback), thus

$$p^* = \frac{a^d + a^s}{b^d + b^s}\,. \tag{7}$$

Assume that price change per instant $d\tau$ is a linear function of the gap between supply and demand through the constant $\kappa > 0$ (a possible specification of κ is given below), representing the price impact of excess demand per unit time $d\tau$ throughout the tâtonnement process

$$\frac{dp(\tau)}{d\tau} = \kappa(q_\tau^d + q_\tau^s)\,. \tag{8}$$

Since the tâtonnement process does not involve exchange of shares throughout the period, the demand and supply functions (5) and (6), need not change. Inserting (5a) and (6a) into (8) yields

$$\frac{dp(\tau)}{d\tau} = \kappa\left(\frac{a^d + a^s}{p(\tau)} - (b^d + b^s)\right). \tag{9}$$

Rewrite (7) as $b^d + b^s = (a^d + a^s)/p^*$, and replace it with $b^d + b^s$ in (9),

$$\frac{dp(\tau)}{d\tau} = \kappa\left(\frac{a^d + a^s}{p(\tau)} - \frac{a^d + a^s}{p^*}\right),$$

thus

$$\frac{dp(\tau)}{d\tau} - \kappa\left(\frac{a^d + a^s}{p(\tau)}\right) = -\kappa\left(\frac{a^d + a^s}{p^*}\right).$$

This equation, together with the initial condition $p_0 \equiv P_t$, being the beginning-of-period price, has a solution given by

$$p(\tau) = p^*\left\{1 + W\left(\left(\frac{p_0}{p^*} - 1\right)e^{-\kappa\frac{a^d + a^s}{(p^*)^2}\tau + \left(\frac{p_0}{p^*} - 1\right)}\right)\right\}, \tag{10}$$

where $W(x)$ is the Lambert W function defined by the implicit function $We^W = x$. As long as the power term $-\kappa\frac{a^d + a^s}{(p^*)^2}\tau + \left(\frac{p_0}{p^*} - 1\right)$ is negative, the intra-period price path converges exponentially to p^*, but if it is positive the price path is divergent. We discuss both cases since both are plausible under reasonable risk aversion parameters and asset allocation states.

Let us consider the convergent case first. The Lambert W function $W(x)$ has a series expansion in power of x, $W(x) = \sum_{n=1}^{\infty}\frac{(-n)^{n-1}}{n!}x^n$, where its linear approximation is applicable for the convergent case.[5] Assume the opening and closing prices are given, one τ increasing. Using the first term of the series of expansion to linearly approximate (10) yields

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[5] This series converges for $|x| < e^{-1} \approx 0.3678$, or in our case, for $\left|\frac{p_0}{p^*} - 1\right| e^{-\kappa\frac{a^d + a^s}{(p^*)^2}\tau + \left(\frac{p_0}{p^*} - 1\right)} < 0.3678$. In order to assure convergence of the series for every τ, substitute $\tau = 0$ and solve for $\left|\frac{p_0}{p^*} - 1\right| e^{\left(\frac{p_0}{p^*} - 1\right)} = 0.3678$ or $We^W = 0.3678$. This

$$p(\tau) = p^* + \left(p_0 - p^*\right) e^{-\kappa \frac{a^d + a^s}{(p^*)^2}\tau + \left(\frac{p_0}{p^*} - 1\right)} + o\left(e^{-\kappa \frac{a^d + a^s}{(p^*)^2}\tau} \right). \tag{11}$$

The linear approximation is justified since the error term $o\left(e^{-\kappa \frac{a^d + a^s}{(p^*)^2}\tau} \right)$ is exponentially small (compared to the leading terms) when τ is increasing. This approximation improves rapidly as τ increases, and for sufficiently large τ it can be considered satisfactory even if $p_0 > 1.278 p^*$ or $0.722 p^* > p_0$ (see footnote 5). It follows from (10) that the convergence rate to the equilibrium price is exponential and the typical system time-scale unit is

$$T = \frac{(p^*)^2}{\kappa (a^d + a^s)} = \frac{p^*}{\kappa} \frac{1}{b^s + b^d}. \tag{12}$$

The system time scale unit characterizes the pace of convergence of a system to its (asymptotic) solution. It is measurable in calendar units and we emphasize that *speed* is inversely related to *time* (T) throughout the tâtonnement process. While convergence is only satisfied at infinity, this measure represents convergence for practical purposes, with a specification of the error term should convergence seize for exogenous reasons prior to infinity. Replacing the original definitions of b^s, b^d, and p^* into the right-most part of (12) yields

$$T = \frac{P_{t+\Delta t}}{\kappa} \frac{1}{N_t - \lambda (N_{a,t} / \delta_a + N_{b,t} / \delta_b)}. \tag{13}$$

The system time scale unit is now defined by the primitives of the asset pricing model and the parameter κ that links between price change and excess demand per unit time. It is easy to verify in terms of comparative statics that a marginal increase in firm size, N_t, which maintains

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yields $W(0.3678) = \frac{p_0}{p^*} - 1$ or $0.722 p^* < p_0 < 1.278 p^*$. Note that based on (10), if $p_0 = p^*$ the price will not change throughout the period as $\frac{p_0}{p^*} - 1 = 0$.

the proportional distribution of shares between both investors (i.e., N_t, $N_{a,t}$, and $N_{b,t}$ are all scaled by a constant factor in excess of unity), reduces *T*, and hence increases the speed of stock price discovery. This empirically testable prediction of our model appears to be new within the context of the CAPM.

In contrast to Asparouhova et al.'s model (2003), where all agents have CARA preferences, the specific structure of heterogeneity in our model has an important effect on the speed of stock price discovery. In particular, assume that firm size N_t remains constant, but one share is reallocated from investor *b* to investor *a*. A unit increase in $N_{a,t}$ coupled by a unit decline in $N_{b,t}$, changes the term $N_{a,t}/\delta_a + N_{b,t}/\delta_b$ in the denominator of *T* by $1/\delta_a - 1/\delta_b$. Because $\delta_a < \delta_b$, this change is positive, which implies that *T* increases and hence the speed of price discovery *declines*. This marginal impact on speed will be stronger as risk preferences are more dispersed. Obviously, the speed of price discovery would increase if type *b* investors were the recipients of the marginal share.

A plausible structure for κ is $\kappa = \beta \dfrac{P_t}{N_t \Delta t}$, where $\beta > 0$. To see why, replace it in (8) and rearrange

$$\frac{dp(\tau)}{P_t} = \beta \frac{d\tau}{\Delta t} \frac{q_\tau^d + q_\tau^s}{N_t},$$

demonstrating that the instantaneous change in price throughout the tâtonnement process is a function of the time interval within Δt and the instantaneous rate of excess demand. It implies that the instantaneous price change at tau, $dp(\tau)/P_t$, is determined by the product of the firm-size adjusted excess demand $(q_\tau^d + q_\tau^s)/N_t$, as it is cleared over time by the fraction $d\tau/\Delta t$ and factored by β. Replacing the assumed structure of κ in (13) yields

$$T = \frac{P_{t+\Delta t}}{\beta P_t} \frac{\Delta t}{1 - \lambda(\pi_{a,t} / \delta_a + \pi_{b,t} / \delta_b)}, \tag{14}$$

where $\pi_{a,t} = N_{a,t} / N_t$ and $\pi_{b,t} = N_{b,t} / N_t$ represent the proportional share distribution between our two investor types at *t*. Note that the denominator of *T* can obtain negative values, representing divergent price paths, an issue we address in detail next. For now, we draw the following conclusions concerning *T* in the convergent case (*T*>0):

1) *T* lengthens (i.e., speed declines) with the percentage price change throughout the period. This implies that low information content would result in a faster speed of price discovery, a prediction of the model which is supported by the empirical findings of Gosnell et al. (1996) and Jennings and Starks (1985).

2) *T* lengthens with λ, the market price for risk, which in turn increases with the equity premium and declines with the variance of stock returns. This implies that, ceteris paribus, the speed of price discovery increases with volatility. This prediction appears to be consistent with the empirical findings of Martens (1998), who found that increased volatility makes the speed of price discovery in LIFFE faster than that of DTB.[6]

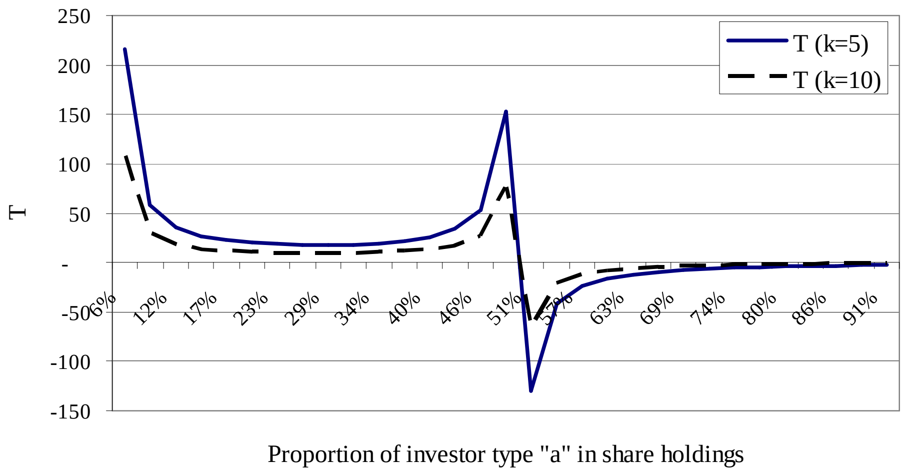
3) The partial derivative of *T* w.r.t. either one of the parameters of relative risk aversion, δ_a and δ_b is negative, implying it is shortened (increasing the speed of price discovery) with a marginal increase in either of them.

4) Based on the Proposition given in Section 3 where $\delta_a << \lambda << \delta_b$, and because $\pi_{b,t} = 1 - \pi_{a,t}$, a marginal change in the distribution of shares between both investor types has non-monotonic effects on *T*.

[6] We mention that Martens compares two exchanges that differ in their capacity to process excess demand, β.

In Figure 1 we present the implications of increasing $\pi_{a,t}$ on the horizontal axis (and hence reducing $\pi_{b,t}$) on *T*. We adjust both risk aversion parameters to secure the consistency between the preferences of the composite investor with the equilibrium price $P_{t+\Delta t}$. We find that *T* obtains high positive values in two distinct cases: first, *T* obtains high values in a highly homogeneous market where $\pi_{a,t} \cong 6\%$, (which implies $\delta_a, \delta_b \to \lambda$), presented on the left-hand-side of Figure 1. Second, *T* obtains high positive values when $\pi_{a,t} \to \left[\frac{\delta_a(\delta_b - \lambda)}{\lambda(\delta_b - \delta_a)}\right]^+$, where $[x]^+$ represents the limit from the left.

Figure 1



System time scale unit as a function of $\pi_{a,t}$

This figure shows that the system time scale unit, *T*, approaches positive infinity in two cases: first, when $\delta_a, \delta_b \to \lambda$, representing a relatively homogenous economy where the composite investor is made up of only $\pi_{a,t}$=6% of type "*a*" investors (having DRRA preferences), and the balance is populated by type "*b*" investors (having CRRA preferences). Second, as $\pi_{a,t}$ approaches (from the left) the critical value presented on the right-hand-side of (15), *T* starts increasing again, and the speed of price discovery declines, albeit still convergent. Once the inequality in (15) holds, *T* turns negative and the price path is divergent. The speed of divergence is faster when the absolute value of *T* is smaller.

As the Figure shows, T obtains relatively low values in-between the two cases. The implication of this on the speed of price discovery is that convergence is very slow under the extreme conditions presented above, but it is faster in-between. The existence of a local minimum implies that a combination of risk preferences and share distribution exists between the two investor-types, which maximizes the speed of price discovery in the market.

We turn now to discuss the case of divergent price paths throughout the tâtonnement process. As noted above, the sign of T depends on the sign of $1-\lambda(\pi_{a,t}/\delta_a+\pi_{b,t}/\delta_b)$ in its denominator. Solving for the condition for a negative denominator we obtain

$$\pi_{a,t} > \frac{\delta_a(\delta_b-\lambda)}{\lambda(\delta_b-\delta_a)}. \tag{15}$$

This means that if the proportion of type a investors, $\pi_{a,t}$, exceeds the term specified on the right, the price path turns from convergent to divergent. This result is also evident in (10), where the power term $-\kappa\frac{a^d+a^s}{(p^*)^2}\tau$ turns positive as a^d+a^s turn negative. Once the inequality in (15) holds and T turns negative, the price discovery path diverges either upward or downward, depending upon whether $p_0 > p^*$ or the opposite, respectively. Because the speed of divergence is inversely related to (the absolute value of) T, it is slow if the inequality in (15) is small, but speed increases the more type a investors are present in the market. In this case, the series approximation (11) might only be valid for small τ and if p_0 is close to p^*; otherwise, it diverges rapidly with τ (until circuit breakers are deployed). In this latter case the relevant solution is given by (10).

5. Calibration and Simulations

The system has been calibrated for post-war US data using the parameters listed in Table 1 for both the convergent and the divergent cases. To distinguish the convergent from the divergent case, we apply different risk preference assumptions, as detailed below.

Table 1

Parameters for Calibration - NYSE

μ	10.0%	σ^2	3.61%
r	2.00%	λ	2.216
σ	19.0%	*Sharpe Ratio*	0.42
$p*$	31.65	Δt	0.004

The parameters in Table 1 were collected from a few sources.[7] Equal endowments were assumed between both investor types (W=100 each) and we solved for risk parameters and the number of shares outstanding that supported the equilibrium price $P^* = 31.65$, being the average transaction closing price at the NYSE in 2004.

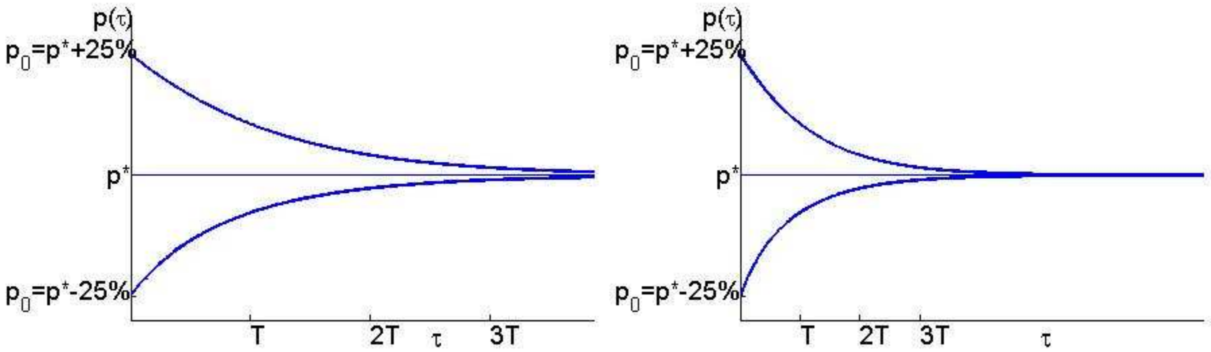
A graphical illustration of the price discovery path is given in Figure 2, where it is assumed on the left chart that the constant κ=5 and the chart on the right was plotted with κ=10. When κ=5 the value that T obtains is 25.972 and when κ=10 T=12.896, half of the former. The risk preferences that were assumed for the convergent case are $\delta_a = 1.618$, $\delta_b = 3.518$ (weighted

[7] The data of postwar US average returns and standard deviation are from Robert Shiller's website http://www.econ.yale.edu/~shiller/data.htm and the average stock price from NYSE Summary Statistics for 2004. Risk aversion parameters are consistent with acceptable measures in the literature, e.g., Cochrane (2001, p. 464), Constantinides (2002), and Mehra and Prescott (1985). The displacement parameter η_b is assumed zero to represent CRRA preferences for type b investors, while the choice of η_a=-42.80 (DRRA preferences) results from adjusting aggregate demand for equities to 10% higher than bonds, being the 2004 average (Federal Reserve Statistical Release, Flow of Funds Accounts of the US, 3, 2006, Table L.100).

average RRA=2.756), and $\eta_a = -42.80,\ \eta_b = 0.00$, consistent with DRRA preferences for investor of type *a* and CRRA preferences for *b*. A smaller κ represents a smaller price impact of excess demand (big stocks, measured by market capitalization).

Figure 2

Left: κ=5 Right: κ=10



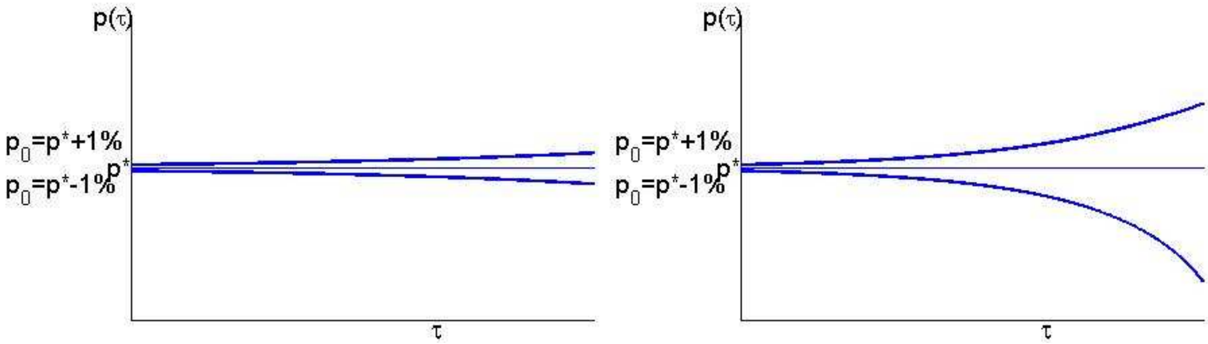
The path and speed of price discovery

The paths of price discovery plotted above demonstrate asymptotic convergence to the equilibrium price after 3-4 *T*s, based on (11). One can also observe that the speed of price discovery is slightly faster for negative rather than positive adjustments.

Using the set of market data presented above but different risk aversion parameters, we simulate two divergent price paths, presented in Figure 3. The divergent price paths are simulated using the risk preferences $\delta_a = 1.351$ and $\delta_b = 6.156$ (weighted average RRA=2.996), while $\eta_a = -42.80$ and $\eta_b = 0.00$. On the left panel κ is small, and the pace of divergence is slow, while the figure on the right represents a higher κ, which implies a faster pace of divergence. In these graphs, *T*=-9.442 for κ=5 and *T*=-4.721 for κ=10. Note that *T* does not appear on the horizontal axis of the divergent case graphs, since the linear series approximation is needed to assess the number of *T*'s until convergence, but in this case the system does not converge. Note further that had the ratio between P^* and P_0 in the figure ($P_{t+\Delta t} / P_t$ in (14)) been greater than 1%, the speed of divergence would have been factored accordingly.

Figure 3

Left: κ =5 Right: κ =10



The path and speed of divergence from one closing price to the next

Divergent price discovery paths are possible if inequality (15) holds, representing the case where the proportion of low RRA investors exceeds the specified threshold. The speed of divergence is faster for negative rather than positive price changes, and materially so for large κ .

6. Summary

This paper develops closed-form measures of the speed of price discovery in a simple asset allocation model with myopic investors, a single riskless asset, and a single risky asset, in a tâtonnement setup. The derivation is facilitated by investor heterogeneity, where heterogeneity is derived in a unique way that implies opposing trading orders (buy/sell) given a price change. The speed of price discovery depends on the primitives of the asset-pricing model, and increases with: 1) the level of average risk aversion, 2) firm size, 3) the asset risk, 4) the price impact of excess demand, and 5) low information content. It is slower when the equity premium increases. The intra-period price path may either converge exponentially to the equilibrium price or exhibit a divergent path.

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