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Mean-Risk Efficiency Analysis of Optimal Investment in Operating Leverage under Real Option Framework

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Abstract

Optimal level of investment in fixed assets also determines the optimal Operating Leverage and the operating risk of the firm. NPV optimization with simultaneous adjustments of the discounting rate to risk is complicated, especially under the assumed Real Options framework that generates asymmetry. Eliminating the real risk by well prices financial hedging may not be available for most cases of real risks. This paper solves the problem by a mean-risk efficiency process that maximizes the slope of a line between a risk free asset and a risky alterative that contains optimal proportions of risky financial asset (market portfolio) and real investment in the firm. The paper also demonstrates the impact of the selected risk criterion on the optimal decision. Under the assumed asymmetrical returns (due to real options) the down-side risk measures such as VaR and Semi-variance perform much better than the conventional mean variance rule.

The analyses shed some insight on the potential conflicts between managers and stockholders.

JEL: D24, D46, D51, G31, G32

Key words: operating leverage, real options, VaR, Semi-Variance, skewness, risk-return efficiency analysis

Introduction

NPV calculations based on Risk Adjusted Cost of Capital (RACC) are the most common means to determine the impact of investment on wealth. However, when risk depends on investment size, the calculation of the optimal investment level should include the impact of investment size on RACC. This specification can be a major obstacle in solving the problem of optimal operating leverage. Lev (1974) presented a theoretical formulation and results of an empirical examination of the positive relationship between β of the stock and the level of Operating Leverage (OL). Ferri and Jones (1979) discussed the empirical negative relationship between financial and operating leverages. Subrahnanyam and Thomaadakis (1980) derived the relationship between systematic risk and various real economy factors including labor-capital ratio. Mandelker and Rhee (1984) formulated the combined impact of Degree of Financial Leverage (DFL) and Degree of Operating Leverage (DOL), and provide empirical evidence for the tradeoff between these two sources of risk. Huffman (1989), and Li and Henderson (1991) further refined the connection between combined leverage and stock risk.

More recently, Wong (2006), and Lederer and Mehta (2005) revisited the issue of evaluating scale-dependent investments which had been examined before in numerous studies.[1] Adopting the real option framework of Kulatilaka (1993) and Kulatilaka and Trgeorgis (1994), Wong (2006) neutralized real investment risks through financial options and forward contracts. Unfortunately, most complicated multi-source real risks can not be hedged by financial

Ledrer and Mehta (2005) calculated NPV of capacity investment using risk-adjusted cash flows or discounting rates, where risk is adjusted according to the market price of risk multiplied by the covariance of the project cash flow and market return. However, Lederer and Mehta (2005) ignore the real option to modify production levels according to revealed prices; they assume that production levels are predetermined by investment size. This paper demonstrates that the real option to change production levels according to revealed prices generates asymmetrical returns that may not be adequately addressed by CAPM risk adjustments.

Similarly to Wong (2006), this paper adopts the real option framework of Kulatilaka (1993), and Kulatilaka and Trgeorgis (1994) and others. Unlike Wong, we do not assume financial hedging of real risks. Wong adopted the limiting assumption of a "bang-bang" production decision, namely, that the firm either abandons production or produces at maximum level. Here, we avoid such a limiting assumption. Due to the asymmetry that is induced by the real option, we also avoid Lederer and Mehta's covariance technique for adjusting discounting rates. Instead, we employ a portfolio efficiency analysis while measuring risk using various downside risk measures rather than variance. In order to reap the pedagogical advantages, the efficiency analysis is divided to the following three stages:[2]

The first step generates the firm's real expected efficient risk–reward tradeoff curve. The decision variable is the size of the investment in the firm. The efficient curve is stated as a function of expected ROI and risk. This stage is relevant for managers or owners whose entire human or financial wealth is invested in the firm.

The second step derives the efficient risk–reward frontier that is generated by diversification; that is, allocating investments between a financial portfolio and the firm at different firm investment intensity levels. This stage identifies the risky frontiers of investors who consider allocating an investment between the firm and a risky financial portfolio.

In the third step, the investor diversifies between the alternative efficient risky portfolios derived in the previous step, and a risk-free asset, in order to determine the optimal combination between financial portfolio investments and alternative investments in capital-intensive firms. The goal of this step is to maximize the risk–reward ratio. According to the separation theorem, this goal is suitable for all investors who have a utility function that is adequately approximated by the assumed function of return and selected risk measure.

Note that the separation theorem, which is a cornerstone in CAPM, holds when down-side risk measures such as VaR or Semi-Variance are used instead of variance.[3] The mean downside risk function has been long considered a better approximation for expected utility maximization than the well-known mean-variance (M-V) function.[4] In case of symmetrical distributions that depend only on the first two moments, the M-V rule approximation is identical to mean down-side risk rules such as Mean-VaR or Mean Semi-Variance. However, in case of asymmetrical distributions, the mean downside risk rules lead to superior performance (Estrada, 2004; Post & Vilet, 2004).

The paper is organized as follows: The first section presents the model under simplifying assumptions. The second section provides a numerical example that demonstrates the basic features of the model. This section provides a comparison between conventional M-V analysis and mean downside rules, and analyzes the impact of separating financial and real decisions, and the effect of the real option to change production levels. The final section offers conclusions.

The Model

Assume an investor who maximizes expected utility by diversifying his investments among three assets, two of which are financially risky. These assets are (1) a financially risk-free asset, (2) a financially risky portfolio such as a market portfolio, and (3) a real investment in the fixed assets of a firm.

The Tailor Expansion of the investor's expected utility around his portfolio mean (μ_1) is:

(1) $EU(\,)=U(\mu_1)+U_2(\mu_1)\cdot \mu_2/2!+U_3(\mu_1)\cdot \mu_3/3!+U_4(\mu_1)\cdot \mu_4/4!+\dots+U_N(\mu_1)\, \mu_{N/N!}+R_{N+1}$

where U_i is the i^{th} derivative of the utility function, μ_i is the i^{th} central moment of the distribution of the returns, and R_{N+1} is the N+1 residual. In the case of normal distributions, M-V as well as mean downside risk functions perfectly approximate expected utility. However, M-V approximation may perform poorly when compared to mean downside risk functions when asymmetry and kurtosis (μ_3 and μ_4) exist.

The real option to change production levels according to the revealed demand and supply function generates μ_3. Thus, it is assumed here that the investor's expected utility can be better approximated by a function of mean downside risk measures such as VaR and Semi-Variance (SV).[5] Namely, the investor selects his/her optimal portfolio – a combination of risk-free assets, risky financial assets, and direct investments in the firm to maximize the slope of the reward-to risk line – a straight line from the risk-free asset to the selected risky portfolio. We show below that in equilibrium, the amount invested in fixed assets is such that enables the investor to obtain the highest attainable reward to risk. Specifically, we replace the well-known Sharp reward-to-variability ratio (RV) by reward-to-VaR (RVR) or reward to semi-variance (RSV), which are defined by:

(2) $RV\equiv(E(R)-R_f)/\sigma$

Where σ stands for the square root of μ_2

(2)' $RVR\equiv(E(R)-R_f)/(VaR)^{0.5}$

(2)'' $RSV\equiv(E(R)-R_f)/(SV)^{0.5}$

where SV stands for semi-variance

Similarly to the well-known Sharp ratio, RV, RVR and RSV are the slopes of a straight tradeoff line between return and risk.[6] The first step in obtaining the highest reward to risk ratio is to develop the mean-risk curve of a real investment in fixed assets of the firm.[7]

The three decision variables of the firm are: level of output Q, level of investment (which determines the level of fixed cost, F), and level of variable cost per unit, which is determined by Q and F and denoted c(Q,F). No constraints are assumed on exogenous minimum levels F and Q or on maximum capacity. However, the variable cost function that will be assumed later, rules out either zero F or infinite production volume.

We assume M&M equilibrium conditions of zero tax and no arbitrage in the debt and equity financial markets. Thus, we do not lose generality by assuming exclusive equity finance. Furthermore, we do not assume Walrassian Arrow-Debreu complete financial markets. Thus, financial markets do not provide full-hedging for all real risks.

At time 0, prices of outputs and inputs are random; consequently, gross margin (GP) per unit is random. However, in order to simplify the presentation, we assume that only the output price is random. Namely,

(3) $GP_j = p_j(Q) - c(Q,F) = p(Q) \cdot (1 + \delta_j) - c(Q,F)$

where $p_j(Q)$ is a non-random demand function for the firm's products, $_j$ is the random demand shock, and δ_j is the jth (j=1,...,K) random shock of the demand function. Let u_{ij} be the probability of obtaining shock δ_j when there is a realization r_i (i=1...N) of the alternative financial portfolio. Thus the probability of the shock δ_j is Σu_{ij}. For the sake of simplicity, it is also assumed that the distribution function δ is independent of both Q and F; the expected value of δ is 0.

At Time 0, the firm selects its production function according to the level of its investment I. The investment determines the annual level of fixed costs F. The investment outflow is determined at Time 0. At Time 1, financial portfolio return r_i and actual prices are revealed, and optimal Q is then determined. Time 2 is a short time after Time 1. At Time 2, output and variable inputs are sold and bought in cash.

Time 2 is also the project termination time. At Time 2, the investor may either sell the un-depreciated portion of the investment with no capital gain or loss, or resume the real investment by reinvesting the depreciated portion of the investment and avoiding the sale of the un-depreciated portion.

Production is set above zero as long as it generates a positive cash flow.

The single cycle time span in our model can be depicted as follows:

	Risky period	Short period of no risk	
-----0----------------------	1-------------------------------	2-------------------------------	→Time line
	Investment outlay	δ and r are revealed Q is determined	Cash from: Output and variable inputs Time 0 for next cycle

Under this model, we compare the mean return-risk efficiency frontiers and optimal decision making, with and without a real option.

If a real option is ignored, the expected operating profit $E(\pi)$ and cash flow E(CF) at Time 2 can be formulated for any selected of Q and F simply as:

(4) $E(CF)= E(\pi)+I(F)=[\ p(Q)-c(Q,F)]\cdot Q-F+I(F)$

In this case, expected cash flow is maximized by optimizing both Q and F simultaneously at Time 0.

Note that according to our simplifying assumption, the fixed cost, F, includes only the depreciation of the investment.[8] Therefore, when we assume at Time 2 that the asset can be sold at cost minus depreciation, the expected terminal cash flow in (4) is simply the expected profit plus the investment I(F).

Under the Real Option (RO) framework, production level at Time 1 is sensitive to the revealed prices and (4) is expanded to (5) as follows:

(5) $E(CF_{RO})= E(\pi_{RO})+I(F)= \Sigma\ [u_j\cdot(\ p(Qj)\cdot(1+\delta_j) -c(Q_j,F))\cdot Q_j]-F+I(F)$

where u_j is the probability of a given demand shock due to all realizations r_i. Namely, $u_j=\Sigma u_{ij}$.

Qj is the selected level of output that we optimize according to the revealed financial return $_j$ at Time 1;

We adopt the following conventional assumptions:

$p'(Q)\leq 0$, $p''(Q)\geq 0$, $c'_Q(Q,F)\geq 0$, $c''_Q(Q,F)\leq 0$, $c'_F(Q,F)\leq 0$, $c''_F(Q,F)\geq 0$.

In order to further simplify the presentation, we also assume a fixed ratio, m, obtains between investments in fixed costs and the periodical fixed cost. Namely,

(6) $I(F)= m\cdot F$

F is not an outflow but only a non-cash item in the P&L statement for the period ending at the end of Time 2. F becomes a cash investment outflow only due to the reinvestment decision at the beginning of the subsequent cycle.

The investor's expected return of his investments in the firm under the real option framework is then:

(7) $E(R_{ro}) = E(CF_{RO})/I = E(\pi_{RO})/m.F + 1$

Note that the maxima of $E(\pi_{RO})$ and $E(CF_{RO})$ are at least above the maxima of $E(\pi)$ and $E(CF)$, respectively. Thus, real options can transform negative expected returns $E(R_{RO})$ to positive ones (Kulatilaka, 1993; Kulatilaka & Trgeorgis, 1994).

Before we continue with the optimization procedure, we use the above notations and definitions to discuss NPV difficulties in solving the specific problem of optimal investment in the firm.

Some difficulties of NPV Valuation

Traditionally the NPV approach is used to optimize investment decisions.[9]

In our model, NPV is:

(8) $NPV = E(CF)/(1+K(F,Q)) - I$

and in case of real options:

(9) $NPV_{RO} = E(CF_{RO})/(1+K_{RO}(F,Q)) - I$

Note that under the optimization procedure, there are two reasons for $NPV_{RO} \geq NPV$. First, as mentioned above, $E(CF_{RO}) \geq E(CF)$. Second, $K_{RO}(F,Q) \leq K(F,Q)$, since the real option hedges downside risk.

The real option to increase Q in response to positive exogenous price shocks is equivalent to holding a call option that increases the gains from a positive price shock; the option to decrease Q in response to a negative price shock is equivalent to holding a covered put option that reduces losses. Thus, the below-mean variance of CF_{RO} is below the lower below-mean variance of CF; the opposite holds for above-mean variances. Namely, the risk of CF_{RO} is lower than the risk of CF even though the variance or β of the real cash flow is not necessarily lower (see example in the next section). Accordingly, it is very possible that an investor who uses variance and β to determine RACC will increase RACC even though the downside risk is lower and RACC should be decreased rather than increased. Such a situation is illustrated with a numerical example in the next section.

Note that in addition to the aforementioned fallacy of the traditional CAPM approach of estimating risk, it seems reasonable to assume that firms do not tend to change RACC according to investment size, and therefore, the NPV approach overvalues (undervalues) projects as investment size increases (decreases).[10]

The real option to change Q generates or increases the asymmetry of profit, while CAPM assumes symmetrical distribution. We demonstrate below that the RO framework generates an asymmetry at a level that depends on the level of the investment in fixed assets. The mean downside risk efficiency analysis presented in the next section captures these asymmetry levels.

Optimal investment according to efficiency analysis approach

Recall that at Time 0, the firm selects an optimal investment that also implies optimal periodical (annual) fixed costs, F*. Given this F*, the firm selects an optimal Qj at Time 1. The first order condition to find F* that maximizes $E(R_{RO})$ is developed in Appendix A. We obtain:

(10) $F^* = -E(GP_{RO})/(\Sigma c'_F \cdot u_j \cdot Q_j)$

where $E(GP_{RO})$ is the expected gross profit when the optimal level of production Q_j is the selected production level when demand shock j occurs.

If expected profit $E(\pi_{RO})$, rather than expected return $E(R_{RO})$, is the maximization criterion, t, instead of (8), we obtain:

(8)' $\Sigma c'_F \cdot u_j \cdot Q_j =$ -1

If there is no real option to change output level at Time 1, (8) is reduced to:

(8)'' $F^* = -(p-c)/ c'_F$

and (8') is reduced to (8''')

(8)''' $c'_F = -1/Q$

The first order conditions for optimizing Q_j for J=1,....,K, involve solutions of the following K equations:

(11) $P_j(Q_j^*) \cdot (1+1/\eta_{dj}) = c(Q_j^*, F_j^*) \cdot (1+1/\eta_{sj})$

where η_{dj} and η_{sj} are the elasticity of demand and supply, respectively, under conditions of shock J.

Note that the K equations in (11) are not modified once profit maximization rather than return maximization is considered.

By comparing the first order conditions with respect to F, we arrive at the following intuitive corollary:

Corollary 1: Expected profit maximization leads to over-investment, compared to expected return maximization, but this tendency to over-invest declines as the proportion of debt increases. The proof is provided in Appendix B.

Note that if we assume that profit maximization is more typical to managers, while return maximization is more typical to investors, risk considerations may change and even reverse the conclusions of Corollary 1. The reason is that managers are typically less diversified than investors and are consequently more sensitive to risk. It is well-known that managers' higher sensitivity to risk leads them to reject investments that are considered desirable by investors. The above is summarized in the following corollary.

Corollary 2: Managers may tend to invest more than investors if they maximize profit and if investors maximize return on investment. The above agency difference in the tendency to invest is (a) reduced when the firm is financed by debt, and (b) reversed when risk is considered and managers are more sensitive to risk.

The above assumptions and optimality conditions also lead to the following intuitive and well-known results.

Corollary 3: The optimal fixed cost F* increases with Q and with expected gross profit margin p-c.

Since $c'_F(Q,F)\leq 0$ and $c''_F(Q,F)\geq 0$, then according to first order conditions with respect to F (see Appendix B), investors who maximize expected return and managers who maximize expected profit both increase optimal F when Q and gross profit margin p-c increase. These results are very intuitive and widely accepted.

Corollary 4: A real option to change Q increases the first and third central moments of the distribution of profits, cash flow and return, while the second central moment (variance) may increase or decrease.

Intuitive proof: The impact of the real option to change Q on the first and third moments is quite clear and has been discussed by us herein above. The impact on the second central moment is proved simply by a numerical example in the next section.

Intuitively, we can prove the corollary as follows:

An option to decrease (increase) Q when gross profit is low (high) is equivalent to holding both put and call options. The put feature of the real option is exercised by lowering production levels when the gross profit margin is lower than expected. Consequently, the lower tail of the profit distribution is trimmed. The call feature of the real option holds when gross profit margin is higher than expected. In that case, production is increased and profit increases. Trimming of the left tail of the profit distribution, t, on one hand, and extending the right tail on the other hand, increases the expected return and also generates positive skewed profits or return. Lowering the left tail of the distribution, while extending the right tail, tends to generate an ambiguous impact on the second moment.

In the next section we extend our analysis to consider risk. Recall that maximizing expected return or profit is unrealistic once risk is considered. F* that maximizes expected return or profit is merely the greatest mean reference point for the more general analysis where risk and financial diversification are considered.

The Efficient Mean-Risk Frontier

F* is the point of maximum expected return without considering risk; this point, however, is not necessarily the point that will be selected by a risk-averse investor. The firm's risk increases with F; therefore, at F>F*, risk is higher and expected return is lower for the firm. Consequently, at the firm level, F is efficient only for all F≤F*. However, this claim may not be correct from the standpoint of an investor who can diversify his investments in assets embodying various risk levels. In the next section, we present a numerical example that demonstrates an optimal selection of F that is much higher than F* that maximizes expected return or profits at the level of the firm.

The procedure of calculating the optimal F that maximizes an expected utility approximation function U=f(E, risk) of an anonymous investor is described below, followed by a graphic exposition and more elaborate explanation of the procedure. The method of exposition is designed to satisfy pedagogical purposes.

- Step 1: Construct a frontier of expected return -risk at the firm level, based on capital intensity levels in the firm (See A-A in Figure 1).

- Step 2: Construct the alternative risky frontiers for different combinations of firm capital intensity levels (A-A) and risky portfolios (i.e., 1-B, 2-B, 3-B in Figure 1).
- Step 3: Select the return-to-risk line with the maximum slope (highest reward to risk). The results ensure that the selection of optimal investment intensity (point 3) is independent of the specific utility function of the investor who can select any point on the reward-to-risk line.

Figure 1 below provides a rough graphical exposition of the above procedure.

[Insert Figure 1 here]

The thickest curve A-A represents the expected return and risk at the firm level derived from the above procedure. Larger investments entail a higher degree of risk. Point 1 on A-A represents the expected return and risk for $I^*=F^*\cdot m$. For all other $I \neq I^*$, the expected return is lower. Since risk increases with I, the efficient mean return-risk frontier at the firm level is limited to Section A1 on A-A. However, when we consider financial diversification, the efficient section may also include also points on A-A to the right of Point 1.

The next step involves generating the efficient risky frontiers from the investor's standpoint, based on each point on the firm's frontier A-A and the risky portfolio whose mean return and risk are depicted by point B. Curves B1, B2 and B3 are three examples of many possible efficient risky frontiers.

The final step combines investments in a riskless asset and in a risky portfolio that includes investment in the firm and investing in the financial risky portfolio. According to the separation theorem, all investors with expected utility approximated by f(E, Risk) should select a risky portfolio that maximizes the reward to risk ratio. In our graphical exposition, the greatest slope (highest reward to risk) is obtained by selecting and connecting Point 3 on A-A with the risky financial portfolio B, giving us O, where RF-R is tangential to O. The selection of Point 3 implies a selection of a specific optimal level of investment in the fixed assets of the firm.

One important element of this procedure is the selection of a risk criterion to improve the approximation of f(E,risk) for the assumed set of admissible utility functions.[11] The asymmetrical return stemming from the real option to change production levels leads us to prefer downside risk measures such as VaR and SV over variance. The numerical example below illustrates the difference between using downside risk measures and variance risk measures.

Numerical Example

1. The Distributions

The risk-free rate r_F is 3%. The return on the risky portfolios, the shocks (delta) on the demand function and the joint and marginal probabilities are given in Table 1.

[Insert Table 1 here]

Variable cost per unit function c is:

(12) $c(Q,F)=a+b\cdot Q^{\eta}+d/F^{\beta}$

Perfect competition is assumed and:

(13) $P=P_0\cdot(1+)$ and $P_0=10$

The basic inputs are provided in Table 2.

[Insert Table 2 here]

2. The Results

The results based on optimization of the reward to VaR ratio are given in the first two lines of Table 3.

[Insert Table 3 here]

The first eight columns represent results at the firm level. When delta is -0.4, Q is 0 and ROI is 4%. When delta increases to -0.2, 0, 0.2, and 0.4, Q increases to 35, 135, 238, and 335, respectively and ROI increases to -3%, 10%, 38%, and 82%, respectively.

Expected ROI is 22% and expected profit is 282. Fixed cost, F* is 52 and thus the amount invested is M·F* or 25·52=1300. The optimal investment in the financial portfolio is zero (see column 9). Columns (10)–(13) represent the reward to risk ratio of the portfolio where risk is calculated according to the various risk measures. Columns (14)–(18) provide some statistics of the portfolio return. When reward to VaR ratio is optimized, the expected return of the portfolio is 21%, the standard deviation (STDEV) is 30%, STDEV below risk-free rate is 15%, and returns are 32% skewed.[12]

When we use other downside risk measures (below risk-free or below mean standard deviations), most results do not change dramatically. When risk is STDEV below mean, F* increases from 52 to 56, and from 52 to 64 when risk is STDEV below R_f. Portfolio parameters and reward to risk results are very close to each other with differences of less than 10%. However, when we use the conventional standard deviation risk measure instead, results are entirely different and, according to most measures, performance is inferior. Compared to optimization using reward to VaR, production levels increase by 35%–114% depending on the specific delta; F* increases from 52 to 80 (54%), and the share of the total investment invested in the financial asset increases from 0 to 11%. The expected ROI declines from 22% to 11%, and expected profit declines from 282 to 228. The expected reward to downside risk measures declines by 46%–66%, whereas the reward to standard deviation increases by no more than 24%. The expected return of the portfolio is much lower (12% instead of 21%) and risk by all measures is also much lower. Recall that Standard deviation risk measure considers deviations toward the positive direction as additional risk.

However, when standard deviation replaces the downside risk measures, the positive skewed returns turn to be close to zero skewed returns (-1%).

Separating managerial and investor decision-making

The potential agency cost of separating managerial and investor portfolio decision-making is exhibited in Table 4. The structure of Table 4 is similar to that of Table 3. The first decision line in the table represents no separation between managerial decisions and investor decisions, where the decision criterion is maximum reward to VaR of the portfolio. In the second case, managers maximize expected profits and investors maximize reward to VaR. In this case, expected profits rise over 60%, from 282 to 450, but F* increases by over 307% from 52 to 160 (+307%), slashing expected ROI from 22% to 11%. When managers maximize expected profits, portfolio has a much lower expected return (12% instead of 21%), standard deviation (14% instead of 30%) and skewness 14% instead of 32%.[13]

[Insert Table 4 here]

Nonetheless, in our example, the agency cost is reflected in excessive investment and excessive production. This phenomenon is eliminated when managers maximize expected ROI or reward to ROI. We must emphasize that this extreme result may not occur in other examples, although we can expect excessive investment and production to decline, but not totally vanish, when managers maximize expected ROI rather than expected profit. When managers consider risk (for example by maximizing reward to VaR of ROI), they reduce investments further, and may invest in the firm an amount that is even smaller than the amount investors would have them invest.

When managers maximize expected profits, and investors maximize reward to standard deviation rather than reward to VaR, the weight of the optimal financial asset increases from zero to 40%. The overall portfolio generates slightly higher expected returns (13% instead of 12%) and standard deviation (15% instead of 14%) but with less skewed returns (11% instead of 14%). This result indicates that reward to standard deviation does not utilize the benefit of skewed ROI returns. Consequently, investors' optimal portfolio contains a greater proportion of financial assets and smaller investments in the firm.

The impact of the real option to change Q

The impact of the real option to change Q is demonstrated in Table 5.

The first row of results in Table 5 shows the results when a real option to change Q exists and when the investor's criterion is maximum reward to VaR.

[Insert Table 5 here]

The first row of results in this table is identical to the results presented in Tables 3 and 4, reflecting the results when a real option to change Q at Time 1 exists, and reward to VaR maximization is considered. The second row presents results in the case that no real option to change Q exists at Time 1. Consequently, Q* and F* are both determined at Time 1 (Q equals 125, and F* increases from 52 to 96). The expected ROI declines from 22% to 9%, while expected profit decreases from 262 to 216. When a real option exists, the optimal portfolio comprises investment in the firm exclusively. When no such option exists, the optimal investment decision is to avoid any investment in the firm and invest 100% of the funds in the financial asset. When reward to VaR criterion is replaced by reward to below-mean standard deviation, and reward to standard deviation, the optimal investment allocations in the financial portfolio drop to 87% and 68%, respectively (see Column 9).

The optimal reward to VaR is 261% when a real option exists, and declines to 85% when no real option to changes production levels at Time 1 exists (See Column 10 in Table 5). Only the optimal reward to standard deviation does not changed significantly, regardless of the optimization criterion or the existence of a real option (and remains in the range of 0.59–0.62, see Column 9 in Table 5). This last result is quite clear: optimization of reward to downside risk measures such as VaR decreases only the left tail of the distribution and allows an increase in its right tail, and therefore, overall variance may not decrease. Indeed, the standard deviation and skewness of the portfolio are highest (30% and 32%, respectively) when a real option exists and reward to VaR optimization is employed. When no real option exists, and reward to standard deviation optimization is used, standard deviation and skewness drop to 13% and 7%, respectively.

The separation of managerial decisions from the investors' decisions also creates a very significant impact when no real option exists. When there is no separation and the optimization criterion is reward to VaR, the level of production is 125 and F* is 96. When decisions are separated, and managers maximize expected profits, production level is 212 and F*=160. However, when managers also consider risk and maximize reward of ROI to VaR, production rises slightly (167 rather than 125) but F* decreases from 96 to 72. This last result is consistent with the well-known claim that managers who are less diversified than the investors tend, due to risk considerations, to reject investments which are efficient for investors.

Conclusion

This paper proposes an efficient mean-risk analysis approach for determining the optimal level of investment in the firm. This approach overcomes some deficiencies of two existing approaches for solving the optimal size of investment in the firm. In the first, the NPV approach, investment size increases the discounting factor that is determined according to CAPM. When a real option to change production levels subsequent to the investment period exists, returns (in terms of ROI or ROE) tend to be more positively skewed. This effect is not well reflected when risk is measured in terms of variance. This paper overcomes this problem by using downside risk measures such as semi-variance and VaR instead of standard deviation. The second approach in the literature which addresses the additional risk due to higher operating leverage calls for the elimination of risk through financial hedging. However, unless very limiting assumptions are imposed, most types of operational risks cannot be effectively hedged. This paper demonstrates both theoretically and through a numerical example how optimal production and investment decisions in the firm and portfolio results are affected by the existence of real options, the specific risk criterion, and the existence of separation between managerial decisions and investors' decisions. A theoretical and/or numerical comparative static of the results should be the next step following this paper.

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